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**Geometric representation of sequences of complexity  $2n + 1$ . (Représentation géométrique de suites de complexité  $2n + 1$ .)** (French) [Zbl 0789.28011](#)

Bull. Soc. Math. Fr. 119, No. 2, 199-215 (1991).

Let  $A$  be a finite set and let  $\Omega = A^{\mathbb{N}}$  be all of the one sided infinite sequences over  $A$ . For any  $u \in \Omega$  and  $n \in \mathbb{N}$  let  $L_n(u)$  be the set of subwords of  $u$  of length  $n$  ( $v \in A^n$  is a subword of  $u$  if for some  $k$  one has  $v_1 = u_{k+1}, \dots, v_n = u_{k+n}$ ), and let  $p_n(u)$  be the cardinality of  $L_n(u)$ . The sequence  $(p_n(u))$  is called the complexity of  $u$ . This paper studies minimal sequences  $u$  in  $\Omega$  of complexity  $n + 1$  and  $2n + 1$ . The analysis uses the de Bruijn graph of  $L_n(u)$  (where two words  $u, v$  are connected if for some  $a, b \in A$  one has  $u = bw$  and  $v = wa$ ). It is shown how all minimal sequences  $u$  of complexity  $p_n(u) = n + 1$  can be described by infinite sequences consisting of two substitutions, and how this leads to (the well-known) isomorphism of the action of the shift on the closed orbit of  $u$  with a rotation on the circle. For a class of the sequences of complexity  $p_n(u) = 2n + 1$  satisfying a regularity condition on their de Bruijn graphs it is shown that these generate closed orbits which are isomorphic to interval exchange transformations with six intervals.

Reviewer: [F.M.Dekking \(Delft\)](#)

**MSC:**

[28D05](#) Measure-preserving transformations  
[11K50](#) Metric theory of continued fractions

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**Keywords:**

minimal sequences; complexity; de Bruijn graph; closed orbits; interval exchange transformations

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