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The group of isometries of a compact Riemannian homogeneous space. (English)

Zbl 0789.53032
Szenthe, J. (ed.) et al., Differential geometry and its applications. Proceedings of a colloquium, held in Eger, Hungary, August 20-25, 1989, organized by the János Bolyai Mathematical Society. Amsterdam: North- Holland Publishing Company. Colloq. Math. Soc. János Bolyai. 56, 597-616 (1992).

Let G be a connected simple compact Lie group and H be a closed Lie subgroup of G. The Killing form of the Lie algebra of G induces a G-invariant Riemannian metric γ_0 on $M = G/H$ called the natural Riemannian metric. Let $I(M) = I(M, \gamma_0)$ be the group of all isometries of γ_0 and $I(M)^0$ be its identity component. Let $\mathrm{Aut}_G M$ be the group of automorphisms of M and $(\mathrm{Aut}_M)^0$ be its identity component, $\mathrm{Sim}_G M$ the group of auto-similitudes of M and $\mathrm{Aut}(G, H)$ be the group of all automorphisms of G mapping H onto itself. The main result of this paper is the following: $I(M) = G(\mathrm{Aut}_G M)^0$ (locally direct product), $I(M) = \mathrm{Sim}_G M = GA$, $A \simeq \mathrm{Aut}(G, H)$, except for the following cases: a) $M = G_2/SU_3 = S^6$, $I(M) = O_7$; b) $M = \mathrm{Spin}_7/G_2 = S^7$, $I(M) = O_8$; c) $M = \mathrm{Spin}_8/G_2 = S^7 \times S^7$, $I(M) = (O_8 \times O_8) \rtimes \langle s\rangle$, s being the transposition of factors; d) $M = G$ with the action l by left translations, γ_0 being the bi-invariant Riemannian metric on G, $I(M) = (\mathrm{Hol}\ G \rtimes \langle s\rangle$, $s : g \to g^{-1}$ for $g \in G$, $\mathrm{Hol}\ G = l(G) \cdot \mathrm{Aut}\ G$.

For the entire collection see [Zbl 0764.00002].

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isometries; automorphisms; auto-similitudes