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Some path properties of iterated Brownian motion. (English) Zbl 0789.60060

Cinlar, E. (ed.) et al., Seminar on stochastic processes, 1992. Held at the Univ. of Washington, DC, USA, March 26-28, 1992. Basel: Birkhäuser. Prog. Probab. 33, 67-87 (1992).

The so-called iterated Brownian motion is the process $Z = (Z(t), t \geq 0)$ defined by $Z(t) = X(Y(t))$, where $(X(t), t \geq 0)$, $(X(-t), t \geq 0)$ and $(Y(t), t \geq 0)$ are three independent standard Brownian motions. The first concern of the paper is to decide whether Z specifies X and Y. The answer is surprisingly nearly positive, in the sense that one can construct from Z two paths $(X'(t), -\infty < t < \infty)$ and $(Y'(t), t \geq 0)$ such that

$$X(t) = X'(t) \text{ for all } t \in (-\infty, \infty) \text{ and } Y(t) = Y'(t) \text{ for all } t \in [0, \infty),$$

or

$$X(t) = X'(-t) \text{ for all } t \in (-\infty, \infty) \text{ and } Y(t) = -Y'(t) \text{ for all } t \in [0, \infty).$$

A similar result also holds when Z is replaced by $X \circ X$. The second concern is an analogue of Khinchin's law of the iterated logarithm. The author shows that a.s.

$$\limsup_{t \to 0+} \frac{Z(t)}{t^{1/4}(\log|\log t|)^{3/4}} = 2^{5/4} 3^{-3/4}.$$

(We mention that this last result can also be deduced from the law of the iterated logarithm for a stable subordinator with index 1/4 due to *L. Breiman* [Ann. Math. Stat. 39, 1818-1824 (1968; Zbl 0192.554)]).

For the entire collection see [Zbl 0780.00020].

Reviewer: J.Bertoin (Paris)

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iterated Brownian motion; Khinchin's law of the iterated logarithm; stable subordinator