

**Ghoussoub, Nassif**

**Duality and perturbation methods in critical point theory.** (English) Zbl 0790.58002

[Cambridge Tracts in Mathematics](#) and [Mathematical Physics](#). 107. Cambridge: Cambridge University Press. xviii, 258 p. (1993).

This rich book develops in a systematic way duality and perturbation methods in order to deal with problems in critical point theory. In particular, it deals with problems where neither ‘the usual compactness condition à la Palais-Smale nor the nondegeneracy conditions à la Fredholm’ are satisfied, by presenting various new variational principles. An aim is to make these methods accessible to non-linear analysts. The following list of the chapters may give an impression of the content (within the text, a wealth of examples is worked out; to name just a few: the Hartree-Fock equation for Coulomb systems, infinite-dimensional Hamilton-Jacobi equations, the forced double pendulum, solutions of elliptic equations involving the critical Sobolev exponent...):

Lipschitz and smooth perturbed minimization problems, Linear and plurisubharmonic perturbed minimization principles, The classical min-max theorem, A strong form of the min-max principle, Relaxed boundary conditions in the presence of a dual set, The critical set in the mountain pass-theorem, Group actions and multiplicity of critical points, The Palais-Smale condition around a dual set – Examples, Morse-indices of min-max critical points, The non degenerate case and the degenerate case, Morse-type information on Palais-Smale sequences +5 Appendices on background material.

This impressive research monograph provides an excellent basis for an advanced course or a seminar on problems of nonlinear analysis. However, I doubt that besides mathematicians it is also ‘accessible to economists and engineers’ as claimed in the cover text. It is written in a quite concise style, for example Ekeland’s variational principle is proven on p. 2.

Reviewer: [F.Colonius \(Augsburg\)](#)

**MSC:**

- [58-02](#) Research exposition (monographs, survey articles) pertaining to global analysis
- [49-02](#) Research exposition (monographs, survey articles) pertaining to calculus of variations and optimal control
- [58E05](#) Abstract critical point theory (Morse theory, Lyusternik-Shnirel’man theory, etc.) in infinite-dimensional spaces

Cited in **3** Reviews  
Cited in **136** Documents

**Keywords:**

[Palais-Smale condition](#); [critical points](#); [duality](#); [perturbation methods](#); [variational principles](#)