**Brighi, B.**; **Chipot, M.**

**Approximation in nonconvex problems.** (English) Zbl 0793.41027

Bandle, C. (ed.) et al., Progress in partial differential equations: calculus of variations, applications. 1st European conference on Elliptic and parabolic problems, Pont-à-Mousson, France, June 1991. Harlow, Essex: Longman Scientific & Technical. Pitman Res. Notes Math. Ser. 267, 150-157 (1993).

Let $\phi : \mathbb{R}^n \to \mathbb{R}$ be a continuous function, $\phi^{**}$ be the convex envelope of $\phi$ i.e. the function defined by $\phi^{**}(\alpha) = \sup\{g(\alpha) : g \text{ convex}, g \leq \phi\}$, $\Omega$ be some bounded, polygonal domain of $\mathbb{R}^n$, $n \geq 1$, with a boundary $\Gamma$, $\tau$ denotes a triangulation of $\Omega$ into simplifices $K$ of mesh size $h = \mathrm{Max}_{K\in\tau}\mathrm{diam}K$, $P_1(K)$ denotes the set of polynomials of degree 1 on $K$ and $V_h^0 = \{v : \Omega \to \mathbb{R}$: $v$ is continuous, $v|_K \in P_1(K)$ $\forall K \in \tau$, $v = 0$ on $\Gamma\}$ ($v|_K$ denotes the restriction of $v$ on $K$). Define the approximated convex envelope of $\phi$ by

$$\phi_h^{**}(\alpha) = \inf_{v\in V_h^0} \left\{ \frac{1}{\Omega} \int_\Omega \phi(\alpha + \nabla v(x))dx \right\}.$$

clearly $\phi_h^{**}(\alpha) \geq \phi^{**}(\alpha)$. In this note, estimates for the difference $\phi_h^{**} - \phi^{**}$ are derived in terms of the mesh size $h$. Restricting to the one dimensional case, the following theorem is proved: Assume that $\lim_{|\xi|\to\infty} \frac{\phi(\xi)}{|\xi|} = +\infty$, $\Omega = (w_-, w_+)$. Denote by $x_0, x_1, \ldots, x_{n+1}$ a subdivision of $\Omega$ such that $w_- = x_0, x_1, \ldots, x_n$, $x_{n+1} = w_+$ and $h = \max_{i=0,\ldots,n} x_{i+1} - x_i$. Then there exists a constant $c = c(\alpha, \phi)$ such that

$$0 \leq \phi_h^{**}(\alpha) - \phi^{**}(\alpha) \leq \frac{c}{|\Omega|} \cdot h$$

where $|\Omega| = w_= - w_-$.

For the entire collection see [Zbl 0780.00014].

Reviewer: T.D.Narang (Amritsar)

**MSC:**

41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)

Cited in **2** Documents

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convex envelope; approximated convex envelope