A positive integer \( n \) is called non-Cayley (\( n \in \text{NC} \)) if there exists a non-Cayley, vertex-transitive graph having \( n \) vertices. This paper investigates the “simplest” case for which only sporadic results are known whether \( n \in \text{NC} \), namely, when \( n = 2pq \) and \( p \) and \( q \) are primes with \( 2 < q < p \). In the first of the two main results, it is shown by construction that \( 2pq \in \text{NC} \) when \( q \mid p - 1 \). Arguing that “the case where there is a vertex-imprimitive group of automorphisms is the heart of the problem”, the authors then prove that if \( q \nmid p - 1 \) but \( p \equiv q \equiv 3 \pmod{4} \) and \( pq \notin \text{NC} \), then any graph on \( 2pq \neq 66 \) vertices admitting a transitive imprimitive group of automorphisms is a Cayley graph. The elegant techniques employed draw extensively from the theory of the classical groups and from combinatorial geometry.

Reviewer: M.E. Watkins (Syracuse)

MSC:

05C25 Graphs and abstract algebra (groups, rings, fields, etc.)
05E15 Combinatorial aspects of groups and algebras (MSC2010)

Keywords:

Cayley graph; imprimitive permutation group; automorphism group; projective unimodular group; vertex-transitive graph

Software:

GAP; GRAPE ; nauty; Cayley

Full Text: DOI

References:
