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A derived algorithm for evaluating $\varepsilon$-expressions over abstract sets. (English) J. Symb. Comput. 15, No. 5-6, 673-704 (1993).

Summary: Four weak theories of pure sets are axiomatically characterized. A decision method is given for checking sentences of the form $\forall y_1 \cdots \forall y_n \exists x \, p$, where $n$ varies over natural numbers and $p$ over unquantified matrices, for provability in each theory. Dually, the method can be used to check $\exists y_1 \cdots \exists y_n \forall x \, \neg p$ for satisfiability. The completeness proof is fully constructive: this means that given a satisfiable constraint of the form $\exists y_1 \cdots \exists y_n \forall x \, \neg p$, a computable model of the axioms which also fulfills the constraint can be synthesized. In this sense, we have a way of automatically generating a concrete representation of the abstract data-type “set” under varying axioms. The problem is also addressed of how to determine $\varepsilon x p$; i.e., how to find a $\zeta$ fulfilling $p(\xi_1, \ldots, \xi_n, \zeta)$ in a computable model $\mathcal{M}$ of one of our theories, as a function of input $\mathcal{M}$-sets $\xi_1, \ldots, \xi_n$. A partial solution to this problem is supplied, which works when $\forall y_1 \cdots \forall y_n \exists x \, p$ is a theorem and $\mathcal{M}$ meets a suitable condition which happens to be satisfied by those models that are produced by our automatic synthesis algorithm. A stronger condition on $\mathcal{M}$ is also characterized that makes $\varepsilon x p$ computable in all cases (at worst through a blind search method). Examples showing the expressive power of the $\exists^* \forall$-constraints and the usefulness of $\varepsilon$-expressions in set computations are included. Envisaged extensions of the proposed methods to axiomatic set theories antithetic to the classical ones are briefly hinted at.

MSC:
03B25 Decidability of theories and sets of sentences
03E30 Axiomatics of classical set theory and its fragments
03B35 Mechanization of proofs and logical operations
03B70 Logic in computer science
68Q60 Specification and verification (program logics, model checking, etc.)
68Q65 Abstract data types; algebraic specification

Keywords:
program synthesis; weak theories of pure sets; decision method; satisfiability; completeness; abstract data-type; computable model; automatic synthesis algorithm; set computations; axiomatic set theories

Software:
ISETL

Full Text: DOI

References:
[1] Aczel, P., (), Lecture Notes