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Remarks on nonlinear Schrödinger equations in one space dimension. (English)

Zbl 0803.35137
Differ. Integral Equ. 7, No. 2, 453-461 (1994).

Summary: We consider the initial value problem for nonlinear Schrödinger equations,

$$i\partial_t u+\frac{1}{2}\partial^2 u=F(u,\partial u,\overline{u},\partial\overline{u}),\quad (t,x)\in\mathbb{R}^+\times\mathbb{R},\qquad u(0,x)=u_0(x),\quad x\in\mathbb{R},\tag{1}$$

where $\partial=\partial_x=\partial/\partial x$ and $F:\mathbb{C}^4\to\mathbb{C}$ is a polynomial having neither constant nor linear terms. Without a smallness condition on the data u_0, it is shown that (1) has a unique local solution in time if u_0 is in $H^{3,0}\cap H^{2,1}$, where

$$H^{m,s}=\left\{f\in S':\|f\|_{m,s}=\left\|(1+x^2)^{s/2}(1-\Delta)^{m/2}f\right\|_2<\infty\right\},\ m,s\in\mathbb{R}.$$

MSC:

35Q55 NLS equations (nonlinear Schrödinger equations)
35Q60 PDEs in connection with optics and electromagnetic theory
35A07 Local existence and uniqueness theorems (PDE) (MSC2000)

Keywords:
initial value problem; unique local solution