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The mean square of the error term for the fourth power moment of the zeta-function.
(English) Zbl 0805.11060
Proc. Lond. Math. Soc., III. Ser. 69, No. 2, 309-329 (1994).

Let

$$\int_0^T \left|\zeta\left(\frac{1}{2}+it\right)\right|^4 dt = Tf(\log T) + E_2(T),$$

where f is an appropriate quartic polynomial. It is shown here that

$$\int_0^T E_2(t)^2 dt \ll T^2(\log T)^C$$

for some constant C. This remarkable result implies the estimates $E_2(T) \ll T^{2/3}(\log T)^C$, and hence $\zeta(\frac{1}{2}+it) \ll t^{1/6}(\log t)^C$, as well as the bound

$$\int_0^T \left|\zeta\left(\frac{1}{2}+it\right)\right|^{12} dt \ll T^2(\log T)^C,$$

with differing values of C. Further theorems describe the mean value of the error terms for $\sum_{n=1}^N d(n)d(n+k)$ and $\sum_{n=1}^{N-1} d(n)d(N-n)$. In particular, the latter has an asymptotic formula with an error term which is $O(N^{\frac{1}{2}+\varepsilon})$ in mean.

The proofs use the spectral theory of the non-Euclidean Laplacian.

Reviewer: D.R.Heath-Brown (Oxford)

MSC:

11M06 $\zeta(s)$ and $L(s,\chi)$
11F72 Spectral theory; trace formulas (e.g., that of Selberg)
11N37 Asymptotic results on arithmetic functions

Keywords:

Riemann zeta-function; fourth power; binary divisor problem; error terms; spectral theory; non-Euclidean Laplacian

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