

Ruiz, Jesús M.; Shiota, Masahiro

On global Nash functions. (English) Zbl 0805.14027

Ann. Sci. Éc. Norm. Supér. (4) 27, No. 1, 103-124 (1994).

There is a number of open problems concerning Nash functions, and a central one is the “separation problem”, which asks whether Nash functions are sufficient to separate the global analytic components of real algebraic sets. The authors introduce a new problem (“equal complexities”): if a semialgebraic set is described by s simultaneous global analytic inequalities, can it be described by s Nash inequalities? The main result of the paper is that “separation” and “equal complexities” are equivalent for compact Nash manifolds. There are other interesting results, such as the fact that “separation” implies “extension” (a Nash function on a Nash subset may be extended to a Nash function on the ambient manifold). The introduction of the paper gives informations about related results in the literature about Nash functions. Since the writing of the paper under review, the authors and the reviewer (to appear in *Am. J. Math.*) have proven that the separation problem has a positive answer on a compact Nash manifold; hence “equal complexities” also hold.

The problem of “equal complexities” is in the line of the works initiated by *L. Bröcker* about the number of inequalities needed to describe semialgebraic sets; the main tool in this activity is the notion of fan, which comes from the algebraic theory of quadratic forms. The main result of the paper is translated in terms of fans, and then the proof uses algebraic tools.

Reviewer: [M.Coste \(Rennes\)](#)

MSC:

[14P20](#) Nash functions and manifolds
[32C07](#) Real-analytic sets, complex Nash functions

Cited in **2** Documents

Keywords:

[separation problem](#); [problem of equal complexities](#); [Nash functions](#); [number of inequalities](#); [fans](#)

Full Text: [DOI](#) [Numdam](#) [EuDML](#)

References:

- [1] C. ANDRADAS , L. BRÖCKER and J. M. RUIZ , Minimal Generation of Basic Open Semianalytic Sets (*Invent. Math.*, Vol. 92, 1988 , pp. 409-430). MR 89f:32016 | Zbl 0655.32011 · Zbl 0655.32011 · doi:10.1007/BF01404461
- [2] C. ANDRADAS , L. BRÖCKER and J. M. RUIZ , Real Algebra and Analytic Geometry , in preparation. · Zbl 0873.14044
- [3] M. ARTIN , On the Solutions of Analytic Equations (*Invent. Math.*, Vol. 5, 1988 , pp. 277-291). MR 38 #344 | Zbl 0172.05301 · Zbl 0172.05301 · doi:10.1007/BF01389777
- [4] L. BERETTA and A. TOGNOLI , Nash Sets and Global Equations (*Bollettino U.M.I.*, Vol. 7, 1990 , pp. 31-44). MR 91b:58001 | Zbl 0735.14037 · Zbl 0735.14037
- [5] J. BOCHNAK , M. COSTE and M.-F. ROY , Géométrie algébrique réelle (*Ergeb. Math.*, Vol. 12, Springer-Verlag, Berlin-Heidelberg-New York, 1987). MR 90b:14030 | Zbl 0633.14016 · Zbl 0633.14016
- [6] J. BOCHNAK and G. EFROYMSON , An Introduction to Nash Functions , in *Géométrie algébrique réelle et formes quadratiques*, pp. 41-54 (*Lecture Notes in Math.*, No. 959, Springer-Verlag, Berlin-Heidelberg-New York, 1982). MR 84b:32013 | Zbl 0516.14019 · Zbl 0516.14019
- [7] N. BOURBAKI , *Commutative Algebra* , Hermann, Paris, 1972 . MR 50 #12997
- [8] L. BRÖCKER , On the Stability Index of Noetherian Rings , in *Real Analytic and Algebraic Geometry*, pp. 72-80 (*Lecture Notes in Math.*, No. 1420, Springer-Verlag, Berlin-Heidelberg-New York, 1990). MR 91g:14055 | Zbl 0696.13011 · Zbl 0696.13011
- [9] L. BRÖCKER , On Basic Semialgebraic Sets (*Expo. Math.*, Vol. 9, 1991 , pp. 289-334). MR 93b:14085 | Zbl 0783.14035 · Zbl 0783.14035
- [10] F. BRUHAT and H. WHITNEY , Quelques propriétés fondamentales des ensembles analytiques réels (*Comment. Math. Helv.*, Vol. 33, 1959 , pp. 132-160). MR 21 #889 | Zbl 0100.08101 · Zbl 0100.08101 · doi:10.1007/BF02565913
- [11] H. CARTAN , Variétés analytiques réelles et variétés analytiques complexes (*Bull. Soc. Math. France*, Vol. 85, 1957 , pp.

- 77-99). Numdam | MR 20 #1339 | Zbl 0083.30502 · Zbl 0083.30502
- [12] M. COSTE and C. DIOP , Problems About Nash Functions , talk in Reelle algebraische Geometrie, Oberwolfach, June 1990 .
- [13] L. VAN DEN DRIES , Model Theory of Fields (Thesis, Utrecht, 1978).
- [14] G. EFROYMSON , Nash Rings in Planar Domains (Trans. Amer. Math. Soc., Vol. 249, 1979 , pp. 435-445). MR 80b:14018 | Zbl 0426.14024 · Zbl 0426.14024 · doi:10.2307/1998801
- [15] G. EFROYMSON , The Extension Theorem for Nash Functions , in Géométrie algébrique réelle et formes quadratiques, pp. 343-357 (Lecture Notes in Math., No. 959, Springer-Verlag, Berlin-Heidelberg-New York 1982). MR 84i:58002 | Zbl 0516.14020 · Zbl 0516.14020
- [16] O. ENDLER , Valuation Theory , Universitext, Springer-Verlag, Berlin-Heidelberg-New York, 1970 . Zbl 0257.12111 · Zbl 0257.12111
- [17] T. Y. LAM , Orderings, Valuations and Quadratic Forms (Reg. Conf. Math., Vol. 52, AMS, 1983). MR 85e:11024 | Zbl 0516.12001 · Zbl 0516.12001
- [18] H. MATSUMURA , Commutative Algebra , second edition (Math. Lecture Note Series, Vol. 56, Benjamin, London-Amsterdam-Tokyo, 1980). MR 82i:13003 | Zbl 0441.13001 · Zbl 0441.13001
- [19] M. NAGATA , Local Rings (Interse. Tracts Math., Vol. 13, John Wiley & Sons, New York-London, 1962). MR 27 #5790 | Zbl 0123.03402 · Zbl 0123.03402
- [20] D. PECKER , On Efroymsen's Extension Theorem for Nash Functions (J. Pure Appl. Algebra, Vol. 37, 1985 , pp. 193-203). MR 87e:58002 | Zbl 0581.14016 · Zbl 0581.14016 · doi:10.1016/0022-4049(85)90097-0
- [21] A. PRESTEL , Pseudo Real Closed Fields , in Set Theory and Model Theory, pp. 127-156 (Lecture Notes in Math., No. 872, Springer-Verlag, Berlin-Heidelberg-New York 1981). MR 84b:12032 | Zbl 0466.12018 · Zbl 0466.12018
- [22] Ch. ROTTHAUS , On the Approximation Property of Excellent Rings (Invent. Math., Vol. 88, 1987 , p. 39-63). MR 88c:14005 | Zbl 0614.13014 · Zbl 0614.13014 · doi:10.1007/BF01405090
- [23] J. M. RUIZ , On Hilbert's 17th Problem and Real Nullstellensatz for Global Analytic Functions (Math. Z., Vol. 190, 1985 , pp. 447-459). Article | MR 87b:32010 | Zbl 0579.14021 · Zbl 0579.14021 · doi:10.1007/BF01215144
- [24] J. M. RUIZ , On the Real Spectrum of a Ring of Global Analytic Functions (Publ. Inst. Recherche Math. Rennes, Vol. 4, 1986 , pp. 84-95). MR 91e:14050 | Zbl 0634.14015 · Zbl 0634.14015
- [25] J. M. RUIZ , A Going-Down Theorem for Real Spectra (J. of Algebra, Vol. 124, No. 2, 1989 , pp. 278-283). MR 91g:14056 | Zbl 0688.14020 · Zbl 0688.14020 · doi:10.1016/0021-8693(89)90130-0
- [26] J. M. RUIZ and M. SHIOTA , Equivalence of Important Problems on Nash Functions (to appear).
- [27] M. SHIOTA , Sur la factorialité de l'anneau des fonctions lisses rationnelles (C. R. Acad. Sci. Paris, Vol. 292, Séries I, 1981 , pp. 67-70. MR 82b:14009 | Zbl 0489.14013 · Zbl 0489.14013
- [28] M. SHIOTA , Nash Manifolds (Lecture notes in Math., No. 1269, Springer-Verlag, Berlin-Heidelberg-New York, 1987). MR 89b:58011 | Zbl 0629.58002 · Zbl 0629.58002
- [29] M. SHIOTA , Extension et factorisation de fonctions de Nash C^∞ (C. R. Acad. Sci. Paris, Vol. 308, Séries I, 1989 , pp. 253-256). MR 90h:32017 | Zbl 0681.32006 · Zbl 0681.32006
- [30] A. TANCREDI and A. TOGNOLI , On the Extension of Nash Functions (Math. Ann., Vol. 288, 1990 , pp. 595-604). MR 92c:32011 | Zbl 0699.32006 · Zbl 0699.32006 · doi:10.1007/BF01444552
- [31] J.-C. TOUGERON , Idéaux de fonctions différentiables (Ergeb. Math., Vol. 71, Springer-Verlag, Berlin-Heidelberg-New York, 1972). MR 55 #13472 | Zbl 0251.58001 · Zbl 0251.58001

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.