Holický, Petr

Čech analytic and almost K-descriptive spaces. (English) Zbl 0806.54030

Czech. Math. J. 43, No. 3, 451-466 (1993).

Recall that a topological space X is Čech-complete if it is a G_δ in some compact (Hausdorff) space, and Čech-analytic if it can be obtained from the Borel sets of X by the Suslin operation (equivalently, Čech-analytic spaces are the completely regular projections of Čech-complete spaces along a separable metrizable factor). A family $\mathcal{U}$ in X is called sb$_d$-σ- decomposable if $U=\bigcup\{U_n : n<\omega\}$ for each $U\in\mathcal{U}$ such that each family $\{U_n : U\in\mathcal{U}\}$ is a disjoint family with a scattered base (equivalently, if $\mathcal{U}$ is point-countable with σ-scattered base). Finally, a space X is almost K-descriptive (resp. almost descriptive) if there is a completely metrizable M and an upper semi-continuous (resp. continuous) compact-valued map $f: M\to X$ which preserves sb$_d$- σ-decomposable families.

The authors show that every Čech-analytic space is almost K- descriptive, and that almost K-descriptive and Čech-analytic coincide with each other (and with analyticity) in metric spaces. Furthermore, they show that the class of almost K-descriptive spaces shares various properties with the Čech-analytic spaces: it is closed under countable unions and intersections, the Suslin operation, closed subspaces, and open subspaces; and an almost K-descriptive completely regular space is σ-scattered or contains a compact perfect set. Finally, it is shown that almost K-descriptive and almost descriptive coincide for subspaces of Banach spaces with the weak topology.

Reviewer: F.van Engelen (Rotterdam)

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54H05 Descriptive set theory (topological aspects of Borel, analytic, projective, etc. sets)

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