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Kodaira-Spencer maps in local algebra. (English) Zbl 0809.13011

Lecture Notes in Mathematics. 1597. Berlin: Springer-Verlag. xvii, 176 p. (1994).

M. Nagata proved in 1955 that in a local ring A for any prime P , such that A/P is analytically unramified, holds $e_0(A_P) \leq e_0(A)$. In 1959 *C. Lech* asked if for any flat local homomorphism $f : (A, \mathfrak{m}) \rightarrow (B, \mathfrak{n})$ the inequality $e_0(A) \leq e_0(B)$ holds, and proved it for some special cases, and in 1964 he asked if there exists an i such that for the sum transform of the Hilbert functions it is true that $H_A^i(n) \leq H_B^i(n)$ for all n , and proved this for $n = 1$, or if the fibre $B/\mathfrak{m}B$ is a complete intersection. In a geometric setting, let $f : (X, x) \rightarrow (Y, y)$ be a flat morphism of Noetherian schemes, is $e_0(Y, y) \leq e_0(X, x)$ and $H_{Y,y}^i \leq H_{X,x}^i$ (coefficientwise)? From this point of view it is natural to look at flat morphisms $f : (X, x) \rightarrow (Y, y)$ with fixed special fibre $(X_y, x) = f^{-1}(y)$ equal to (X_0, x_0) . The author has shown (without flatness assumption) that $H_{Y,y}^1 H_{X_y,x}^0 \geq H_{X,x}^1$. If f is tangentially flat, i.e. the induced morphism $df : C(X, x) \rightarrow C(Y, y)$ of tangent spaces is flat, there is an equality $H_{Y,y}^1 H_{X_y,x}^0 = H_{X,x}^1$, which gives $H_{Y,y}^1 \leq H_{X,x}^1$. If Schlessinger's T^1 of the tangent cone has no elements of degree less than -1 , all deformations are tangentially flat, which enables the author to find many classes of singularities satisfying the inequality $H_{Y,y}^0 \leq H_{X,x}^0$.

One main aim with this monograph is to prove a somewhat weaker version of the inequality, namely

$$H_{Y,y}^1 H_{X_y,x}^0 = H_{X,x}^1 \prod_{d=2}^{\infty} ((1 - T^d)/(1 - T))^{n(d)}$$

for residually separable flat morphisms. Here $n(d) = \dim T_{C(X_y,x)}^1(-d)$. In order to do so, the author has to carefully study more general filtrations than the usual ones, so-called Artin-Rees filtrations.

Reviewer: R.Fröberg (Stockholm)

MSC:

- 13D40 Hilbert-Samuel and Hilbert-Kunz functions; Poincaré series
- 13D10 Deformations and infinitesimal methods in commutative ring theory
- 13-02 Research exposition (monographs, survey articles) pertaining to commutative algebra
- 14B12 Local deformation theory, Artin approximation, etc.
- 14A05 Relevant commutative algebra
- 16S80 Deformations of associative rings
- 13H15 Multiplicity theory and related topics

Cited in **1** Review
Cited in **3** Documents

Keywords:

Kodaira-Spencer maps; transform of Hilbert functions; tangentially flat morphisms; deformations; Artin-Rees filtrations

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