Barth, W.; Nieto, I.
Abelian surfaces of type (1,3) and quartic surfaces with 16 skew lines. (English)

The purpose of this article is to relate the geometry of certain moduli spaces of complex abelian surfaces with some quartic surfaces containing a certain configuration of lines. In particular, one of these moduli spaces is shown to be a Calabi-Yau threefold. Part of these results were also independently found by I. Naruki [Proc. Japan Acad., Ser. A 67, No. 7, 223-225 (1991; Zbl 0763.14019)]. The constructions go as follows. It was shown in Nieto’s thesis [see I. Nieto, “Invariante Quartiken unter der Heisenberg Gruppe T” (Erlangen 1989; Zbl 0668.14028)] that the Kummer surface of a general abelian surface with a type (2,6) polarization embeds (after blowing-up its 16 singular points) as a smooth quartic \( Q \) in \( \mathbb{P}^3 \), invariant under the order-16 Heisenberg group \( H \). The 16 blown-up half-periods become 16 skew lines on \( Q \), which form an orbit under the action of \( H \).

There is a second \( H \)-orbit of 16 lines of \( Q \), corresponding to symmetric divisors \( \Theta \) such that \( 2\Theta \) represents the polarization. In particular, this construction yields a birational correspondence between the moduli space of abelian surfaces with a symmetric line bundle of type (1,3) and a level-2 structure, and a certain subvariety \( M \) of the Grassmannian of lines in \( \mathbb{P}^3 \) (those belonging to a 16-tuple as above). This map is \( H \)-equivariant and induces a birational correspondence between the moduli space of abelian surfaces with a polarization of type (1,3) and a level-2 structure, and the quotient \( M/H \). Here \( M/H \) parametrizes the sets of 16 lines as above. Since each quartic \( Q \) contains two such sets, \( M/H \) is an unbranched double cover of the variety \( N \) that parametrizes the quartics \( Q \) as above.

Explicit equations are given: in \( \mathbb{P}^5 \), \( M \) is defined by \( \sum x_i^2 = \sum 1/x_i^2 = 0 \) and \( N \) by \( \sum x_i = \sum 1/x_i = 0 \). These allow the authors to prove that \( M \) is of general type, and that \( M/H \) is birational to a smooth Calabi-Yau threefold with Euler characteristic 80. Similarly, \( N \) is birational to a smooth Calabi-Yau threefold with Euler characteristic 100, whose Hodge numbers were calculated by Van Straten.

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MSC:

- 14J10 Families, moduli, classification: algebraic theory
- 14K10 Algebraic moduli of abelian varieties, classification
- 14J30 3-folds

Keywords:

- moduli spaces of complex abelian surfaces; quartic surfaces; configuration of lines; Calabi-Yau threefold