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On a class of elliptic systems in $\mathbb{R}^n$. (English) Zbl 0809.35020

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Summary: We consider a class of variational systems in $\mathbb{R}^N$ of the form

$$-\Delta u + a(x)u = f(x,u,v), \qquad -\Delta v + b(x)v = g(x,u,v)$$

where $a, b : \mathbb{R}^N \to \mathbb{R}$ are continuous functions which are coercive; i.e., $a(x)$ and $b(x)$ approach plus infinity as x approaches plus infinity. Under appropriate growth and regularity conditions on the nonlinearities $F_u(\cdot)$ and $F_v(\cdot)$, the (weak) solutions are precisely the critical points of a related functional defined on a Hilbert space of functions u, v in $H^1(\mathbb{R}^N)$.

By considering a class of potentials $F(x,u,v)$ which are nonquadratic at infinity, we show that a weak version of the Palais-Smale condition holds true and that a nontrivial solution can be obtained by the generalized mountain pass theorem. Our approach allows situations in which $a(\cdot)$ and $b(\cdot)$ may have negative values, and the potential $F(x,s)$ may grow either faster or slower than $|s|^2$.

MSC:

35J60 Nonlinear elliptic equations

35J50 Variational methods for elliptic systems

Keywords:

weak Palais-Smale condition; generalized mountain pass theorem

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