Neeb, Karl-Hermann

A convexity theorem for semisimple symmetric spaces. (English) Zbl 0809.53058

Pac. J. Math. 162, No. 2, 305-349 (1994); correction ibid. 166, No. 2, 401 (1994).

Let τ be an involution of a connected semi-simple Lie group G with Lie algebra $\mathfrak{g}$, and let G^τ (resp. $\mathfrak{h} = \mathfrak{g}^\tau$) be the set of fixed points $g \in G$ (resp. $X \in \mathfrak{g}$) of τ. Using the Cartan involution θ of G satisfying $\theta \circ \tau = \tau \circ \theta$, and denoting $K = G^\tau$, the author first constructs subalgebras $\mathfrak{m}$, $\mathfrak{a}$, $\mathfrak{n}$ (resp. subgroups M, A, N) of $\mathfrak{g}$ (resp. G) having the following properties: (i) $\mathfrak{g} = \mathfrak{h} + \mathfrak{m} + \mathfrak{a} + \mathfrak{n}$, (ii) $G^\tau M_0 AN$ is an open subset of G, (iii) $G^\tau \cap MAN = G^\tau \cap M$, and (iv) $\mathfrak{h} \cap (\mathfrak{m} + \mathfrak{a} + \mathfrak{n}) = \mathfrak{h} \cap \mathfrak{m}$. Next, the author shows that for $g = hman \in G$ with $h \in G^\tau$, $m \in M$, $a \in A$, $n \in N$ the element $\log a \in \mathfrak{a}$ is well-defined. Thus, a map $L : G^\tau MAN \to \mathfrak{a}$ can be defined by $L(g) = \log a$. Now, an open subgroup H of G^τ is called essentially connected if $H = H_0 Z_{K\cap H}(\mathfrak{a})$ holds, where Z means the centralizer. An element $a \in A$ is called admissible if $aH \subset G^\tau MAN$ holds. The author proves the following convexity theorem: Let $H \subset G^\tau$ be essentially connected and let $a \in A$ be admissible. Then $L(aH) = \mathrm{conv}(W.\log a) + C(a)$ holds, where W is the Weyl group derived from the root system Δ defined by the maximal Abelian subalgebra $\mathfrak{a}$ in $\{X \in g \mid \tau X = -X, \theta X = -X\}$ and where $C(a)$ is a certain convex cone in $\mathfrak{a}$. The author also succeeds in describing the set $\log(A_{adm})$, where A_{adm} denotes the set of all admissible elements $a \in A$.

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22E46 Semisimple Lie groups and their representations

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convexity theorem

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