zbMATH Open

THE FIRST RESOURCE FOR MATHEMATICS

Yamamoto, Masahiro

Conditional stability in determination of densities of heat sources in a bounded domain.

(English) Zbl 0810.35032

Desch, W. (ed.) et al., Control and estimation of distributed parameter systems: nonlinear phenomena. International conference in Vorau (Austria), July 18-24, 1993. Basel: Birkhäuser. ISNM, Int. Ser. Numer. Math. 118, 359-370 (1994).

Summary: We consider the heat equation in a bounded domain $\Omega \subset \mathbb{R}^r$:

$$\frac{\partial u}{\partial t}(x,t) = \Delta u(x,t) + \sigma(t) f(x) \qquad (x \in \Omega,\ 0 < t < T),$$

$$u(x,0) = 0 \quad (x \in \Omega), \qquad \frac{\partial u}{\partial n}(x,t) = 0 \quad (x \in \partial\Omega,\ 0 < t < T).$$

Assuming that σ is a known function with $\sigma(0) \neq 0$, we prove: (1) $f(x)$ $(x \in \Omega)$ can be uniquely determined from the boundary data $u(x,t)$ $(x \in \partial\Omega,\ 0 < t < T)$. (2) If f is restricted to a compact set in a Sobolev space, then we get an estimate:

$$\|f\|_{L^2(\Omega)} = O\Big(\big(\log \tfrac{1}{\eta}\big)^{-\beta}\Big) \quad \text{as} \quad \eta \equiv |u(\cdot,\cdot)\|_{H^1(0,T;L^2(\partial\Omega))} \downarrow 0.$$

Here the exponent β is given by the order of the Sobolev space which is assumed to contain the set of f's.

For the entire collection see [Zbl 0801.00053].

MSC:

35K05 Heat equation
35R25 Ill-posed problems for PDEs
35R30 Inverse problems for PDEs
93B30 System identification

Keywords:

conditional stability; boundary observation; density of heat source