Grigor'yan, Alexander
Heat kernel upper bounds on a complete non-compact manifold. (English) Zbl 0810.58040
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Let $\Lambda(\nu)$ be a positive continuous decreasing function defined for $\nu \in \mathbf{R}_+$ and let M be a smooth connected noncompact geodesically complete Riemannian manifold of dimension $n \geq 2$. One says that a Λ-isoperimetric inequality is valid for a domain $\Omega \subset M$ if, for any subdomain $D \subset \Omega$ the first Dirichlet eigenvalue $\lambda_1(M)$ for the Laplacian on M is bounded below by $\Lambda(\text{Vol } D)$. The author's main result is that the existence of such a Λ-isoperimetric inequality for a precompact domain $\Omega \subset M$ implies a strong upper bound for the heat kernel $p(x,y,t)$ on M, which in turn implies a lower bound for the k-th eigenvalue of the Laplacian on M. The upper bound for the heat kernel is in terms of a function $V(t)$ defined by $t = \int_0^{V(t)^-} (\nu\Lambda(\nu))^{-1} d\nu$. Much of the work involves the growth properties of the function V. The author's estimate $p(x,y,t) \leq C(V(ct))^{-1} \exp(-r^2/Dt)$ for $t > 0$, positive constants c, C and $D > 4$ arbitrarily close to 4, includes many previously known estimates, including those of Varopoulos, Cheng, Li and Yau, and Davies. For further details see the paper.

Reviewer: J.S.Joel (Kelly)

MSC:

58J35	Heat and other parabolic equation methods for PDEs on manifolds
35P15	Estimates of eigenvalues in context of PDEs
58J50	Spectral problems; spectral geometry; scattering theory on manifolds

Keywords:
heat kernel; isoperimetric inequalities; spectral geometry

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