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A class of analytic functions defined by fractional derivation. (English) Zbl 0813.30016

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Let $T = T_p(A, B, p^{-1}\alpha, \beta, \lambda)$ denote the class of p-valent functions, which have the form

$$f(z) = z^{-p} - \sum_{n=1}^{\infty} a_{p+n} z^{p+n},\ z \in U = \{z : |z| < 1\},\ a_{n+p} \geq 0,\ n \in \mathbb{N},$$

and satisfy the condition

$$\left| \frac{\Omega_z^{(\lambda,p)} f(z) - 1}{B\Omega_z^{(\lambda,p)} f(z) - [B + (A - B)(1 - p^{-1}\alpha)]} \right| < \beta \quad \text{for } z \in U,$$

where $0 \leq p^{-1}\alpha < 1$, $0 < \beta \leq 1$, $0 \leq \lambda \leq 1$, $-1 \leq A \leq 1$, $0 < B \leq 1$ and

$$\Omega_z^{(\lambda,p)} = \frac{\Gamma(1 + p - \lambda)}{\Gamma(1 + p)} z^{\lambda - p} D_z^{\lambda} f(z),$$

where $D_z^{\lambda} f$ is the fractional derivative operator of order α [see f.e. *S. Owa*, Fractional calculus, Proc. Workshop, Ross Priory, Univ. Strathclyde/Engl. 1984, Res. Notes Math. 138, 164-175 (1985; Zbl 0614.30014)].

In this paper some results concerning the radii of p-valently close-to- convexity, starlikeness and convexity for the class T are obtained. Also some classes preserving integral operator of the form

$$F(z) = \frac{c + p}{z^c} \int_0^z t^{c-1} f(t) dt, \quad c > -p,$$

for the class T are determined.

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MSC:

30C45 Special classes of univalent and multivalent functions of one complex variable (starlike, convex, bounded rotation, etc.)

30C75 Extremal problems for conformal and quasiconformal mappings, other methods

Keywords:
p-valent functions; fractional derivative

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