Brézis, Haïm

Convergence in $\mathcal{D}'$ and in L^1 under strict convexity. (English) Zbl 0813.49016

Lions, Jacques-Louis (ed.) et al., Boundary value problems for partial differential equations and applications. Dedicated to Enrico Magenes on the occasion of his 70th birthday. Paris: Masson. Res. Notes Appl. Math. 29, 43-52 (1993).

Let $\Omega \subset \mathbb{R}^n$ be a bounded open set and let (u_n) be a sequence in $L^1(\Omega; \mathbb{R}^n)$ which converges "weakly" to some limit $u \in L^1(\Omega; \mathbb{R}^n)$. Let $\mathcal{J} : \mathbb{R}^n \to \mathbb{R}$ be a convex function such that

$$\limsup_{n\to\infty} \int_\Omega \mathcal{J}(u_n) \leq \int_\Omega \mathcal{J}(u). \tag{1}$$

The assumption that (u_n) converges to u weakly in L^1 (that is, for the weak $\sigma(L^1, L^\infty)$ topology) is a very restrictive assumption and it is desirable to replace it by the much weaker and more natural assumption that (u_n) converges to u in the sense of distributions

$$u_n \to u \quad \text{in} \quad D'(\Omega; \mathbb{R}^n). \tag{2}$$

The main theorem of this paper is the following:

Let (u_n) be a sequence in $L^1(\Omega; \mathbb{R}^n)$ and let $u \in L^1(\Omega; \mathbb{R}^n)$ be such that (1) and (2) hold. Assume that $\mathcal{J}$ is strictly convex. Then $u_n \to u$ strongly in $L^1_{\text{loc}}(\Omega; \mathbb{R}^n)$. If, in addition, we suppose that $\lim_{|t|\to\infty} \mathcal{J}(t) = +\infty$, then $u_n \to u$ strongly in $L^1(\Omega; \mathbb{R}^n)$.

For the entire collection see [Zbl 0782.00097].

Reviewer: L.Mikołajczyk (Łódź)

MSC:

49J45 Methods involving semicontinuity and convergence; relaxation

Keywords:

sequence converging in the sense of distributions; strict convexity; strong convergence