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Extinction of contact and percolation processes in a random environment. (English)

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Summary: We consider the (inhomogeneous) percolation process on $\mathbb{Z}^d \times \mathbb{R}$ defined as follows: Along each vertical line $\{x\} \times \mathbb{R}$ we put cuts at times given by a Poisson point process with intensity $\delta(x)$, and between each pair of adjacent vertical lines $\{x\} \times \mathbb{R}$ and $\{y\} \times \mathbb{R}$ we place bridges at times given by a Poisson point process with intensity $\lambda(x, y)$. We say that (x, t) and (y, s) are connected (or in the same cluster) if there is a path from (x, t) to (y, s) made out of uncut segments of vertical lines and bridges.

If we consider only oriented percolation, we have the graphical representation of the (inhomogeneous) d-dimensional contact process. We consider these percolation and contact processes in a random environment by taking $\delta = \{\delta(x);\ x \in \mathbb{Z}^d\}$ and $\lambda = \{\lambda(x, y);\ x, y \in \mathbb{Z}^d,\ \|x - y\|_2 = 1\}$ to be independent families of independent identically distributed strictly positive random variables; we use δ and λ for representative random variables. We prove extinction (i.e., no percolation) of these percolation and contact processes, for almost every δ and λ, if δ and λ satisfy $\mathbb{E}\{(\log(1 + \lambda))^\beta\} < \infty$ and $\mathbb{E}\{(\log(1 + 1/\delta))^\beta\} < \infty$ for some $\beta > 2d^2(1 + \sqrt{1 + 1/d} + 1/2d)$, and if $\mathbb{E}\{(\log(1 + \lambda/\delta))^\beta\}$ is sufficiently small.

MSC:

60K35 Interacting random processes; statistical mechanics type models; percolation theory

Keywords:

percolation process; contact process; contact processes in a random environment

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