Ninomiya, Hirokazu

**Some remarks on inertial manifolds.** (English) Zbl 0815.35037

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The aim of the author is to construct inertial manifolds for a class of evolution equations of the form (1) $u_t + Au + R(t,u) = 0$ which takes place on a Hilbert space $H$ with norm $\|\ \|$. In order to assure the existence of such manifolds the author imposes some assumptions on (1) which are to some extent weaker than those usually encountered. $A$ is assumed to be a positive selfadjoint operator with compact resolvents and spectrum $0 < \lambda_1 \le \lambda_2 \dots$; we denote by $w_j$ an eigenvector of $\lambda_j$. Crucial to the theory is a spectral gap condition of the form (2) $\lambda_{N+1} - \lambda_N > K_1(\lambda_{N+1}^{\gamma/2} + \lambda_N^{\gamma/2})^2$ for some $N > 0$. The constant $K_1$ and $\gamma \in (0, \frac{1}{2})$ are determined by the nonlinearity $R(t,u)$ which is assumed to satisfy the following conditions:

(3) $\|R(t,u)\| \le K_0$, (4) $\|R(t,u) - R(t,v)\| \le K_1\|A^\gamma(u-v)\|$, (5) $\|R(t+h,u) - R(t,u)\| \le K_2|h|$,

where $u, v \in \mathrm{dom}(A^\gamma)$. Under these assumptions the evolution equation (1) generates a semiflow $S(t,t_0)$ which maps $\mathrm{dom}(A^\gamma)$ into itself and which has some smoothness properties which are proved in the course of the paper. Let $P_N$ be the orthogonal projection onto $\mathrm{span}(w_1, \dots, w_N)$. The first main result (theorem 1) states that there is a Lipschitz mapping $\varphi(\cdot,\cdot)$ from $\mathbb{R} \times P_N\mathrm{dom}(A^\gamma)$ into $(1-P_N)\mathrm{dom}(A^\gamma)$ such that for each $t \in \mathbb{R}$ the manifold $M_t = \mathrm{graph}(\varphi(t,\cdot))$ has the following properties: (a) $M_t = S(t,t_0)M_{t_0}$, $t, t_0 \in \mathbb{R}$, (b) if $u(t) = S(t,t_0)u_0$ then there exists $v_0 \in M_{t_0}$ such that

$$\left\| A^\gamma\big(S(t+t_0,t_0)u_0 - S(t+t_0,t_0)v_0\big)\right\| \le C_1 e^{-t\nu},\ t \ge 0$$

for suitable $C_1$, $\nu > 0$ independent of $t_0, t$. There are further results of perturbational nature which compare the manifolds $M_t$ with $\widetilde{M}_t$, $t \in \mathbb{R}$ which emerge from nonlinearities $R(t,u)$ and $\widetilde{R}(t,u)$ respectively, and which loosely speaking express that $M_t$ and $\widetilde{M}_t$ are asymptotically close if $R(t,u)$ and $\widetilde{R}(t,u)$ are asymptotically close. The proofs are based on a series of technical lemmas. Among these, lemma 3.3 in particular expresses that the flow $S(t,t_0)$ has a certain squeezing property which differs from that usually encountered.

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**MSC:**

35G10 Initial value problems for linear higher-order PDEs
35K25 Higher-order parabolic equations
35B40 Asymptotic behavior of solutions to PDEs
58D25 Equations in function spaces; evolution equations
47H20 Semigroups of nonlinear operators



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