

Muzychuk, Mikhail

On association schemes of the symmetric group S_{2n} acting on partitions of type 2^n . (English)

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Let $(G; X)$ be a transitive permutation group on a finite set X and R_0, \dots, R_d the 2-orbits of $(G; X)$, i.e. the orbits of the induced action of G on $X \times X$. Then the pair $(X, \{R_i\}_{i=0}^d)$ is an association scheme (see *E. Bannai* and *T. Itô* [Algebraic combinatorics. I: Association schemes (1984; Zbl 0555.05019)]). This scheme is called 2-orbit scheme of $(G; X)$ and is denoted by $2\text{-orb}(G; X)$. For example, the Johnson scheme $J(m, n)$ is introduced as a 2-orbit scheme of the symmetric group S_n acting on the m -element subsets of $\{1, 2, \dots, n\}$. The structure of fusion schemes of $J(m, n)$ was studied by many authors. Since $J(m, n)$ is a 2-orbit scheme of the primitive permutation representation of S_n , it is natural to study other primitive representations of these groups. Let S_{2n} be the symmetric group on the set $\{1, 2, \dots, 2n\}$. Consider the induced action of S_{2n} on the set P_n of all partitions of $\{1, 2, \dots, 2n\}$ into n equal parts of size 2. In this paper, the author investigates the schemes $2\text{-orb}(S_{2n}; P_n)$. In particular, he proves that there is no non-trivial fusion scheme of $2\text{-orb}(S_{10}; P_5)$.

Reviewer: [A.Kondrat'ev \(Ekaterinburg\)](#)

MSC:

05E30 Association schemes, strongly regular graphs

20B30 Symmetric groups

20B25 Finite automorphism groups of algebraic, geometric, or combinatorial structures

Cited in 2 Documents

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partitions; association scheme; 2-orbit scheme; Johnson scheme; symmetric group; fusion schemes; primitive permutation representation