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A converse to Stanley's conjecture for Sl_2. (English) Zbl 0817.13004

Proc. Am. Math. Soc. 121, No. 1, 47-51 (1994).

Let $G = Sl(V)$ where V is a two-dimensional vector space over an algebraically closed field k of characteristic zero. Define $W = \bigoplus_{i=1}^m S^{d_i}V$, $d = \dim W = \sum(d_i + 1)$, and $R = SW$, where SW denotes the symmetric algebra of W. Define for $n \geq 0$, $s^{(n)} = n + (n-2) + \cdots + 1 = \frac{(n+1)^2}{4}$ if n is odd, $s^{(n)} = n + (n-2) + \cdots + 2 = \frac{n(n+2)}{4}$ if n is even, and put $s = \sum_{i=1}^m s^{(d_i)}$. It follows from a conjecture of *R. P. Stanley* [Proc. Symp. Pure Math. Am. Math. Soc., Columbus 1978, Proc. Symp. Pure Math. 34, 345-355 (1979; Zbl 0411.22006)] that $(R \otimes S^\mu V)^G$ is Cohen-Macaulay if $\mu < s - 2$. This conjecture was proved in almost complete generality by the author [J. Am. Math. Soc. 2, No. 4, 775-799 (1989; Zbl 0697.20025)].

B. Broer proved [Indag. Math., New Ser. 1, No. 1, 15-25 (1990; Zbl 0703.15031)] a partial converse to Stanley's conjecture for Sl_2. In this note we will prove a complete converse.

MSC:

13C14 Cohen-Macaulay modules
14L24 Geometric invariant theory
13A50 Actions of groups on commutative rings; invariant theory

Keywords:

Cohen-Macaulayness of invariant modules for reductive algebraic groups

Full Text: DOI