Bolthausen, Erwin

Localization of a two-dimensional random walk with an attractive path interaction. (English)

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Ann. Probab. 22, No. 2, 875-918 (1994).

Let $(X_t)_{t\geq 0}$ be the time-continuous, symmetric random walk on $\mathbb{Z}^d$, starting from $X_0 = 0$ with generator

$$Af(x) = \frac{1}{2} \sum_{y:|y-x|=1} \big(f(y) - f(x)\big).$$

Let S_T be the set of points in $\mathbb{Z}^d$ visited by X_t up to time $T > 0$ and set $N_T =$ the cardinality of the set S_T. Define a new distribution $d\widehat{P}_T$ by $d\widehat{P}_T = \exp[-N_T]dP/E\exp[-N_T]$. For $x \in \mathbb{Z}^d$ and $r > 0$, put $B_x(r) = \{y \in \mathbb{Z}^d : [y - x| \leq r\}$. The main result is a description of the shape of S_T in the two-dimensional case. That is, when $d = 2$, for any $\varepsilon > 0$, one has

$$\lim_{T\to\infty} \widehat{P}_T \left(\bigcup_{x \in B_0(\rho T^{1/4})} \left\{ B_x\big(\rho(1-\varepsilon)T^{1/4}\big) \subset S_T \subset B_x\big(\rho(1+\varepsilon)T^{1/4}\big) \right\} \right) = 1,$$

where $\rho = (\lambda/\pi)^{1/4}$ and λ is the principal eigenvalue of $-\Delta/2$ in the ball of radius 1. The proof of the result requires a refinement of the analysis of Donsker and Varadhan. Partial discussion is also given to higher dimensions.

Reviewer: Chen Mu-fa (Beijing)

MSC:

60F10 Large deviations
60K35 Interacting random processes; statistical mechanics type models; percolation theory
60J25 Continuous-time Markov processes on general state spaces

Keywords:

large deviation; asymptotic behavior; random walk

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