Alinhac, Serge
Blowup for nonlinear hyperbolic equations. (English) Zbl 0820.35001

Progress in Nonlinear Differential Equations and their Applications. 17. Boston: Birkhäuser. xiv, 112 p. (1995).

These lecture notes give a short and clear introduction to the problem of singularity formation in nonlinear hyperbolic problems, by one of the active researchers in this rapidly growing field. Coverage includes typical blow-up criteria, estimates of the maximal time of existence by asymptotic methods, but first and foremost general strategies for establishing the blow-up mechanism. Indeed, recent work has shown that singularity formation can be understood by performing appropriate changes of variables acting on dependent and independent variables; the singularities of the solution are simply due to the properties of these changes of variables. These techniques form the unifying theme of the book. They enable one to go, for the first time with this level of generality, beyond estimates of the time of the first singularity. This new methodology seems to have permanent value.

The style is both lively and rigorous, and the proofs are well-organized making the main ingredients transparent. The examples are concrete and well-chosen. The material is at the forefront of current research, and the open problems are clearly introduced.

One may mention that the questions mentioned in the book relative to the reviewer's work have been answered since the book was written: the results do extend to the non-analytic case, and they do describe the correct blow-up mechanism for data close to constants.

As the author mentions, his work does not aim to be encyclopaedic; it nevertheless is a very good snapshot of the subject. This book is very stimulating and enjoyable. It should encourage students and researchers alike to contribute to the subject.

Reviewer: S.Kichenassamy (Minneapolis)

MSC:

35-02	Research exposition (monographs, survey articles) pertaining to partial differential equations
35L67	Shocks and singularities for hyperbolic equations
35B20	Perturbations in context of PDEs
35L70	Second-order nonlinear hyperbolic equations
35A20	Analyticity in context of PDEs

Cited in **84** Documents

Keywords:

singularity formation; nonlinear hyperbolic problems; blow-up criteria