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Algebraic $\mathcal{D}$-modules and representation theory of semisimple Lie groups. (English)
Zbl 0821.22005
Eastwood, Michael (ed.) et al., The Penrose transform and analytic cohomology in representation theory. AMS-IMS-SIAM summer research conference, June 27 - July 3, 1992, South Hadley, MA, USA. Providence, RI: American Mathematical Society. Contemp. Math. 154, 133-168 (1993).

This is an interesting, readable and fairly self-contained exposition describing a new approach to the classification of the irreducible Harish-Chandra modules using the techniques of algebraic geometry, based in part on the work of *H. Hecht, W. Schmid, J. Wolf* and the author [cf. e.g. Invent. Math. 90, 297-332 (1987; Zbl 0699.22022)]. In a broad perspective this approach can be viewed as a far-reaching generalization of the classical realization of irreducible representations of compact semi-simple Lie groups described by the theorem of Borel-Weil.

The principal tool employed in the paper is the localization functor of Beilinson and Bernstein which provides means for establishing the equivalence of the category of $\mathcal{U}(\mathfrak{g})$-modules with an infinitesimal character with the category of $\mathcal{D}$-modules on the flag variety of $\mathfrak{g}$. This in turn induces an equivalence of the category of Harish-Chandra modules with an infinitesimal character with a category of Harish-Chandra sheaves on the flag variety.

Following this scheme the author describes in the first part of the paper the basic notions and constructions of the algebraic theory of $\mathcal{D}$- modules and also presents needed results on the structure of K-orbits in the flag variety of $\mathfrak{g}$. Subsequently a classification of all irreducible Harish-Chandra sheaves and a necessary and sufficient condition for vanishing of cohomology of irreducible Harish-Chandra sheaves is given [cf. the author, Harmonic analysis on reductive groups, Prog. Math. 101, 209-222 (1991; Zbl 0760.22019)]. This leads to a geometric classification of irreducible Harish-Chandra modules in a final part of the paper. The paper closes with a comparison of the given classification with the Langlands classification and a detailed elaboration of the construction for the case of the group $SU(2,1)$.

For the entire collection see [Zbl 0780.00026].

Reviewer: A.Strasburger (Warszawa)

MSC:

22E46	Semisimple Lie groups and their representations
32C38	Sheaves of differential operators and their modules, D-modules

Cited in **6** Documents

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irreducible Harish-Chandra modules; compact semi-simple Lie groups; localization functor; infinitesimal character; $\mathcal{D}$-modules; flag variety; irreducible Harish-Chandra sheaves; Langlands classification