Holub, James R.

Pre-frame operators, Besselian frames, and near-Riesz bases in Hilbert spaces. (English) Zbl 0821.46008

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Summary: A problem of enduring interest in connection with the study of frames in Hilbert space is that of characterizing those frames which can essentially be regarded as Riesz bases for computational purposes or which have certain desirable properties of Riesz bases. In this paper, we study several aspects of this problem using the notion of a pre-frame operator and a model theory for frames derived from this notion. In particular, we show that the deletion of a finite set of vectors from a frame $\{x_n\}_{n=1}^{\infty}$ leaves a Riesz basis if and only if the frame is Besselian (i.e., $\sum_{n=1}^{\infty} a_n x_n$ converges $\Leftrightarrow (a_n) \in \ell^2$).

MSC:

46B15 Summability and bases; functional analytic aspects of frames in Banach and Hilbert spaces

46C05 Hilbert and pre-Hilbert spaces: geometry and topology (including spaces with semidefinite inner product)

47A53 (Semi-) Fredholm operators; index theories

Keywords:

Besselian frames; frames in Hilbert space; characterizing those frames which can essentially be regarded as Riesz bases; pre-frame operator; model theory for frames

Full Text: DOI

References:

[1] I. Daubechies, Frames of coherent states, phase space localisation, and signal analysis (to appear).

[2] Ingrid Daubechies, A. Grossmann, and Y. Meyer, Painless nonorthogonal expansions, J. Math. Phys. 27 (1986), no. 5, 1271 – 1283. · Zbl 0608.46014 · doi:10.1063/1.527388

[3] R. J. Duffin and A. C. Schaeffer, A class of nonharmonic Fourier series, Trans. Amer. Math. Soc. 72 (1952), 341 – 366. · Zbl 0049.32401

[4] Nelson Dunford and Jacob T. Schwartz, Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space, With the assistance of William G. Bade and Robert G. Bartle, Interscience Publishers John Wiley & Sons New York-London, 1963. · Zbl 0128.34803

[5] Christopher Heil, Wavelets and frames, Signal processing, Part I, IMA Vol. Math. Appl., vol. 22, Springer, New York, 1990, pp. 147 – 160. · Zbl 0721.46006 · doi:10.1007/978-1-4684-6393-$4_1$1

[6] C. Heil, Wiener amalgam spaces in generalized harmonic analysis and wavelet theory, Ph.D. Dissertation, University of Maryland, College Park, MD, 1990.

[7] A. J. E. M. Janssen, Bargmann transform, Zak transform, and coherent states, J. Math. Phys. 23 (1982), no. 5, 720 – 731. · Zbl 0486.46027 · doi:10.1063/1.525426

[8] J. R. Retherford and J. R. Holub, The stability of bases in Banach and Hilbert spaces, J. Reine Angew. Math. 246 (1971), 136 – 146. · Zbl 0207.12101

[9] Robert Schatten, Norm ideals of completely continuous operators, Second printing. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 27, Springer-Verlag, Berlin-New York, 1970. · Zbl 0090.09402

[10] Martin Schechter, Principles of functional analysis, Academic Press, New York-London, 1971. · Zbl 0211.14501

[11] Ivan Singer, Bases in Banach spaces. I, Springer-Verlag, New York-Berlin, 1970. Die Grundlehren der mathematischen Wissenschaften, Band 154. · Zbl 0198.16601

[12] J. Zak, Finite translations in solid state physics, Phys. Rev. Lett. 19 (1967), 1385-1397.