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Solution of the inverse problem of the calculus of variations. (English) Zbl 0821.49026

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This paper has the same title as the well-known paper of *J. Douglas* [Trans. Am. Math. Soc. 50, 71-128 (1941; Zbl 0025.18102)]. In the last one only regular first order variational integrals were discussed. This paper proposes a geometrical method based on infinitely prolonged Monge systems together with a far going generalization of Poincaré-Cartan (PC) forms, which is available in the nonregular case even for all constrained variational problems.

The reasonings are carried out in the space of variables $x, y^1, \dots, y^m, z^1, \dots, z^m$ ($m \geq 1$) endowed with the contact forms $\vartheta^i \equiv dy^i - z^i dx$ and the Lagrange density $\lambda = f dx$. One can verify that there is a unique PC form $\xi = \lambda + \sum f_i \vartheta^i$ ($f_i \equiv \partial f / \partial z^i$) satisfying

$$\xi \cong \lambda,\ d\xi \cong 0 \pmod{\vartheta^1, \dots, \vartheta^m}.$$

An inverse problem means that if the extremals of $\int \lambda$ are given in advance, then one searches for the relevant density λ or equivalently, for the relevant PC form ξ. In more detail, a regular inverse problem means that the vector field F (tangent to the extremals) is given and one searches a form ξ of the special kind $\mathcal{A} : \xi = f dx + \sum f_i \vartheta^i$ satisfying moreover $\mathcal{B} : \det(f_{ij}) \neq 0$ (the regularity, $f_{ij} \equiv \partial^2 f / \partial z^i \partial z^j$) and $\mathcal{C} : F \rfloor d\xi = 0$. Author's idea may be explained as follows. If $d\xi$ is already known, then ξ is determined up to a total differential and the Lagrange density $\lambda = f dx$ up to a divergence. The form $d\xi$ can be determined if the restriction $\widetilde{d\xi}$ on a fixed hyperplane $x =$ const. is known. In order to determine $\widetilde{d\xi}$, a Monge system, including a modified Lie derivative, is to be resolved. An alternative approach to the inverse problem by employing the first integrals in a different and more direct manner is also discussed.

This method is suitable in the non-regular case and to the inverse problem for all constrained variational integrals

$$\int p^* \lambda \to \text{ extremum}, \quad p^* \omega \equiv 0 \quad (\omega \in \Omega)$$

where $p = p(t)$ are curves (mappings of an interval $a \leq t \leq b$ into the underlying space) and Ω consists of all contact forms. It avoids the use of Lagrange multipliers. The concluding part is concerned with various topics, in particular the geodesics field theory and the 23rd Hilbert's problem are mentioned.

Reviewer: S.Shih (Tianjin)

MSC:

49N45 Inverse problems in optimal control

Keywords:

Poincaré-Cartan form; Lagrange problem; Monge systems; inverse problem; constrained variational integrals

Full Text: EuDML