Wiles, Andrew

Modular elliptic curves and Fermat's Last Theorem. (English) Zbl 0823.11029

Ann. Math. (2) 141, No. 3, 443-551 (1995).

The main result is the proof of the Taniyama-Weil conjecture for a large class of elliptic curves over $\mathbb{Q}$. These include semistable curves, and thus the result implies the famous Fermat conjecture.

To achieve this one shows that in many cases the Hecke algebra of a modular curve is the base of a universal deformation of the associated p-adic Galois representation. Here $p \geq 3$, and the representation modulo p must be irreducible. If this holds for $p = 3$, then everything follows from results of Langlands-Tunnell, as $\mathrm{PGL}(2, \mathbb{F}_3) \cong S_4$ is solvable. If the mod 3 representation is reducible, one can use $p = 5$ (and the result for $p = 3$).

In the meantime there has been more progress, extending the result to elliptic curves with semistable reduction at 3 and 5. The restriction stems from the argument above, and limitations of our present crystalline techniques.

The contents in more detail.

Chapter I introduces the universal deformation ring, various local conditions on representations, and the corresponding tangent spaces. These are H^1's of certain cohomology theories, and the corresponding Euler characteristic is computed using the results of Tate- Poitou.

Chapter II treats Hecke algebras. It is shown that they are Gorenstein (this comes down to multiplicity one), and Ribet's theory of change of level is used to start the reduction to the minimal case. A key fact is always that certain Hecke operators are redundant in the definition of the Hecke algebra. This is easy for primes of good reduction, but involved for the others.

Chapter III brings the introduction of certain auxiliary primes $q \equiv 1 \bmod p^n$ which are also very important in the subsequent paper of Taylor-Wiles [Ann. Math. (2) 141, No. 3, 553–572 (1995; Zbl 0823.11030)]. It then reduces the assertion to the fact that the Hecke algebra is a complete intersection. That this condition holds is the content of Taylor-Wiles.

Chapter IV treats the dihedral case. This does not occur for semistable curves, and requires the techniques of Kolyvagin-Rubin.

Chapter V actually proves the Taniyama-Weil conjecture for many elliptic curves.

An appendix explains the relevant commutative algebra.

Reviewer: G.Faltings (Bonn)

MSC:

11G05	Elliptic curves over global fields
11F11	Holomorphic modular forms of integral weight
11D41	Higher degree equations; Fermat's equation
14H52	Elliptic curves

Keywords:

Fermat's last theorem; proof of the Taniyama-Weil conjecture; elliptic curves; semistable curves; Hecke algebra of a modular curve; p-adic Galois representation

Full Text: DOI