Cavaliere, M. P.; Rossi, M. E.; Valla, G.
The strong Castelnuovo lemma for zerodimensional schemes. (English) ⟨Zbl 0824.14041⟩
Orecchia, Ferruccio (ed.) et al., Zero-dimensional schemes. Proceedings of the international conference held in Ravello, Italy, June 8-13, 1992. Berlin: de Gruyter. 53-63 (1994).

Let $\mathbb{P}^n$ be the projective space of dimension n over an algebraically closed field of characteristic zero. A finite subscheme X in $\mathbb{P}^n$ is said to be in linearly general position if for every proper linear subspace $L \subset \mathbb{P}^n$ one has $\deg(X \cap L) \leq 1 + \dim(L)$. If one cuts a reduced, irreducible, nondegenerate projective variety V of codimension n in $\mathbb{P}^r$ with a general linear subspace of dimension n, one gets a set X of distinct points on this $\mathbb{P}^n$ in linear general position. This explains why this notion has a central role in many problems of algebraic geometry, especially in the so-called Castelnuovo theory as developed by Castelnuovo and more recently by *J. Harris* (with the collaboration of *D. Eisenbud*) ["Curves in projective space", Sém. Math. Supér. 85 (1982; Zbl 0511.14014)] and *M. L. Green* [J. Differ. Geom. 19, 125-171 (1984; Zbl 0559.14008)]. A classical result in this field is the so-called Castelnuovo lemma which states that any set $X \subset \mathbb{P}^n$ of $d \geq 2n + 3$ points in linearly general position, which imposes a most $2n + 1$ conditions on the system of quadrics in $\mathbb{P}^n$, lies on a rational normal curve. In the quoted paper, M. Green showed a more subtle result, the so-called strong Castelnuovo lemma, for any set $X \subset \mathbb{P}^n$ of points in linearly general position.

Continuing recent works of D. Bayer, D. Eisenbud and J. Harris, leading to extend Castelnuovo theory to more general finite subschemes of the projective space, in the present paper the authors extend the proof of the strong Castelnuovo lemma for zero-dimensional schemes in linearly general position from the reduced to the nonreduced case.

For the entire collection see [Zbl 0797.00007].

Reviewer: V.Di Gennaro (Roma)

MSC:

14M05	Varieties defined by ring conditions (factorial, Cohen-Macaulay, semi-normal)
14C20	Divisors, linear systems, invertible sheaves
14A15	Schemes and morphisms
13D40	Hilbert-Samuel and Hilbert-Kunz functions; Poincaré series

Cited in **3** Documents

Keywords:

linearly general position; strong Castelnuovo lemma; zero-dimensional schemes