To Fu Ma

A note on the existence of two nontrivial solutions of a resonance problem. (English)

Zbl 0824.35038
Port. Math. 51, No. 4, 517-523 (1994).

Existence of two nontrivial solutions of a semilinear problem at resonance is proved in this paper. The problem $-\Delta u = \lambda_1 u + g(x,u)$ in G, $u = 0$ on ∂G is studied, where G is a smooth bounded domain in $\mathbb{R}^N$, Δ is the usual Laplacian, λ_1 is the first eigenvalue of $-\Delta$ and $g(x,u)$ is a Carathéodory function such that $g(x,0) = 0$, $|g(x,s)| \leq a|s|^p + b$, with $a, b > 0$ and $0 < p < (N+2)/(N-2)$ if $N \geq 3$, and $|G(x,s)| \leq k(x)$ (with $G(x,s) = \int_0^s g(x,t)dt$) for some $k \in L^1(G)$. If, moreover, $\lim_{s\to 0} G(x,s)/s^2 = m(x)$ in the $L^1(G)$ sense with $m \geq 0$, $\int_G \limsup_{|s|\to\infty} G(x,s)dx \leq 0$ and $G(x,s) \leq (\lambda_2 - \lambda_1)s^2/2$ for all s (here λ_2 is the second eigenvalue) then the problem has (at least) two nontrivial solutions. The method of proof is variational: the associated functional is C^1 on $H_0^1(G)$ and bounded from below, but it is not coercive. However, it is possible to show that the Palais-Smale condition holds on some interval and this together with a deformation lemma allows to conclude by using minimax arguments.

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MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces
35J20 Variational methods for second-order elliptic equations

Keywords:

two nontrivial solutions; semilinear problem at resonance; Palais-Smale condition; minimax arguments

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