Joly, J. L.; **Metivier, G.**; **Rauch, J.**
Nonlinear high frequency hyperbolic waves. (English) Zbl 0824.35077
Murthy, M. K. V. (ed.) et al., Nonlinear hyperbolic equations and field theory. Papers from a workshop on nonlinear hyperbolic equations held in Varenna, Italy, 1990. Harlow: Longman Scientific & Technical. Pitman Res. Notes Math. Ser. 253, 121-143 (1992).

Our work on oscillations is devoted to a rigorous investigation of the interaction of high frequency nonlinear waves for hyperbolic partial differential equations. We seek formal solutions of the nonlinear system

$$A_0(x,u)\partial_0 u + A_1(x,u)\partial_1 u + F(x,u) = 0.$$

Here $x=(x_0,x_1)\in\mathbb{R}^2$, $u=(u_1,u_2,\dots,u_N)$ is a real vector valued function and A and F are assumed to be smooth matrix and vector valued functions respectively. On a neighborhood of the solutions considered, the system is assumed to be strictly hyperbolic with first variable x_0 timelike.

The point of departure is the asymptotic expansion of linear geometric optics,

$$u^\varepsilon \sim \sum_{k\geq 0} \varepsilon^k U_k\big(x, \varphi(x)/\varepsilon\big) \text{ as } \varepsilon \to 0.$$

Here $U_k(x,\theta)$ is periodic or almost periodic in θ, and the phase φ is real with $d\varphi \neq 0$ and satisfies the eikonal equation which asserts that the points $(x, d\varphi(x))$ belong to the characteristic variety. The linear interaction of two such waves yields a superposition

$$\sum_{k\geq 0} \varepsilon^k U_k\big(x, \varphi_1(x)/\varepsilon\big) + \sum_{k\geq 0} \varepsilon^k W_k\big(x, \varphi_2(x)/\varepsilon\big).$$

When nonlinear equations are considered one finds nonlinear expressions of the form $F(U(x,\varphi_1/\varepsilon), W(x,\varphi_2/\varepsilon))$. This suggests the expansions

$$u^\varepsilon \sim \sum_{k\geq 0} \varepsilon^k U_k\big(x, \varphi_1(x)/\varepsilon,\ \varphi_2(x)/\varepsilon, \dots, \varphi_r(x)/\varepsilon\big)$$

for nonlinear problems. The profiles $U_k(x,\theta_1,\theta_2,\dots,\theta_r)$ are assumed to be smooth and either periodic or almost periodic with respect to the θ variables.

The amplitude of the high frequency oscillations is crucial. If the amplitude is too large, the exact solutions do not exist on a domain independent of ε. If the amplitude is too small, the principal term of the expansion will not display effects. The hypothesis of weakly nonlinear geometric optics places one exactly on the boundary.

For the entire collection see [Zbl 0780.00036].

MSC:

35L70 Second-order nonlinear hyperbolic equations
35C20 Asymptotic expansions of solutions to PDEs

Keywords:

formal solutions; linear geometric optics; weakly nonlinear geometric optics