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**Contact geometry and linear differential equations.** (English. Russian original) Zbl 0826.58037
Russ. Math. Surv. 48, No. 3, 93-132 (1993); translation from Usp. Mat. Nauk 48, No. 3(291), 97-134 (1993).

The authors present a method for studying differential equations based on contact geometry of the phase space. Their principal objects are the homogeneous phase space, homogeneous Lagrangian manifolds, the homogeneous canonical Maslov operator, Fourier-Maslov integral operators (FMIO's) and so on. But they formulate everything in the language of contact geometry, where they consider the cotangent bundle without zero section factorized by the group $R_*$ of non-zero real numbers. This gives a finer description of singularities and enables the authors to obtain a quite elegant and transparent formula for the Fourier transform of homogeneous functions.

The paper is organized as follows. In §1 the authors present certain geometrical questions having to do with symplectic and contact geometry, and also establish the so called classification lemma, which deals with the one-to-one correspondence between Lagrange manifolds and defining functions. In §2 they develop the Fourier transform for homogeneous functions and the third section is an exposition of FMIO's in homogeneous situations. In §4 they apply this theory to problems of discontinuity propagation and to the study of lacunae of Green's function. An important application of contact geometry is to be found in theorems dealing with the microlocal classification of Hamiltonians. These questions are considered in §5. Finally, in §6 the techniques of §3 and the results of §5 are used to study solubility and to prove finiteness theorems for operators of principal type, i.e. operators whose principal symbol has no contact stationary points.

Reviewer: J.Eichhorn (Greifswald)

**MSC:**

58J40 Pseudodifferential and Fourier integral operators on manifolds
37J55 Contact systems
58J60 Relations of PDEs with special manifold structures (Riemannian, Finsler, etc.)
53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)

**Keywords:**

Fourier integral operators; contact geometry; phase space

**Full Text:** DOI