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**Decoupling inequalities for the tail probabilities of multivariate $U$- statistics.** (English)

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Summary: We present a decoupling inequality that shows that multivariate $U$- statistics can be studied as sums of (conditionally) independent random variables. This result has important implications in several areas of probability and statistics including the study of random graphs and multiple stochastic integration. More precisely, we get the following result: Let $\{X_j\}$ be a sequence of independent random variables on a measurable space $(\mathcal{S}, S)$ and let $\{X_i^{(j)}\}$, $j = 1, \dots, k$, be $k$ independent copies of $\{X_i\}$. Let $f_{i_1 i_2 \dots i_k}$ be families of functions of $k$ variables taking $(S \times \cdots \times S)$ into a Banach space $(B, |\cdot|)$. Then, for all $n \geq k \geq 2$, $t > 0$, there exist numerical constants $C_k$ depending on $k$ only so that

$$P\left(\Big|\sum_{1\leq i_1\neq i_2\neq\cdots\neq i_k\leq n} f_{i_1\dots i_k}\left(X_{i_1}^{(1)}, X_{i_2}^{(1)}, \dots, X_{i_k}^{(1)}\right)\Big| \geq t\right)$$
$$\leq C_k P\left(C_k\Big|\sum_{1\leq i_1\neq i_2\neq\cdots\neq i_k\leq n} f_{i_1\dots i_k}\left(X_{i_1}^{(1)}, X_{i_2}^{(2)}, \dots, X_{i_k}^{(k)}\right)\Big| \geq t\right).$$

The reverse bound holds if, in addition, the following symmetry condition holds almost surely:

$$f_{i_1 i_2 \dots i_k}(X_{i_1}, X_{i_2}, \dots, X_{i_k}) = f_{i_{\pi(1)} i_{\pi(2)} \dots i_{\pi(k)}}(X_{i_{\pi(1)}}, X_{i_{\pi(2)}}, \dots, X_{i_{\pi(k)}}),$$

for all permutations $\pi$ of $(1, \dots, k)$.

**MSC:**

60E15 Inequalities; stochastic orderings
60D05 Geometric probability and stochastic geometry



**Keywords:**
multivariate $U$-statistics; decoupling inequality; multiple stochastic integration

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