Ben-David, Shai; **Cesa-Bianchi, Nicolò**; **Haussler, David**; **Long, Philip M.**
Characterizations of learnability for classes of $\{0, \ldots, n\}$-valued functions. (English)
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Summary: We investigate the PAC learnability of classes of $\{0, \ldots, n\}$- valued functions ($n < \infty$). For $n = 1$ it is known that the finiteness of the Vapnik-Chervonenkis dimension is necessary and sufficient for learning. For $n > 1$ several generalizations of the VC- dimension, each yielding distinct characterization of learnability, have been proposed by a number of researchers. In this paper we present a general scheme for extending the VC-dimension to the case $n > 1$. Our scheme defines a wide variety of notions of dimension in which all these variants of the VC-dimension, previously introduced in the context of learning, appear as special cases. Our main result is a simple condition characterizing the set of notions of dimension whose finiteness is necessary and sufficient for learning. This provides a variety of new tools for determining the learnability of a class of multi-valued functions. Our characterization is also shown to hold in the "robust" variant of PAC model and for any "reasonable" loss function.

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