Vaughan, Jerry E.

A countably compact, separable space which is not absolutely countably compact. (English)
Zbl 0833.54012
Commentat. Math. Univ. Carol. 36, No. 1, 197-201 (1995).

A space X is absolutely countably compact provided for every open cover $\mathcal{U}$ of X and for every dense subspace $Y \subset X$ there exists a finite subset $A \subset Y$ such that $St(A,\mathcal{U}) = X$ (if we remove "for every dense subspace $Y \subset X$" and write $A \subset X$ instead of $A \subset Y$, then we obtain a weaker condition, starcompactness, which is known to be equivalent to countable compactness in the class of Hausdorff spaces). Answering a question of the reviewer, the author constructs a space having the properties in the title. Also, he gives an example of a countably compact topological group which is not absolutely countably compact. Both examples are derived from the following interesting observation: if a T_1 space X has an open cover $\mathcal{U}$ which does not have a finite subcover, then the product space $X^{\mathfrak{k}}$, where $\mathfrak{k} = |\mathcal{U}|$, is not absolutely countably compact.

Reviewer: M.V.Matveev (Moskva)

MSC:

54D20 Noncompact covering properties (paracompact, Lindelöf, etc.)
54B10 Product spaces in general topology
54G20 Counterexamples in general topology
54H11 Topological groups (topological aspects)

Keywords:

absolutely countably compact space; starcompactness; countable compactness; countably compact topological group

Full Text: EuDML