

[Adimurthi](#); [Mancini, G.](#)

The Neumann problem for elliptic equations with critical nonlinearity. (English)

[Zbl 0836.35048](#)

Ambrosetti, A. (ed.) et al., Nonlinear analysis. A tribute in honour of Giovanni Prodi. Pisa: Scuola Normale Superiore, Quaderni. Università di Pisa. 9-25 (1991).

The authors consider the semilinear elliptic boundary value problem in a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 3$:

$$\Delta u + \alpha(x)u = u^p, \quad u > 0 \text{ in } \Omega, \quad u = 0 \text{ on } \Gamma_0, \quad \partial u / \partial n = 0 \text{ on } \Gamma_1, \quad \partial \Omega = \Gamma_0 \cup \Gamma_1,$$

where $p = (n+2)/(n-2)$ is the critical exponent. They prove existence of at least one nontrivial solution in the following cases:

- 1) $\alpha(x) \equiv \lambda > \lambda_1$, where λ_1 is a sufficiently large constant, and $\Gamma_0 = \emptyset$, $\Gamma_1 = \partial \Omega$;
- 2) $\alpha \in L^\infty$ such that $-\Delta + \alpha$ is a positive operator on $\{u \in H^1(\Omega) \mid u = 0 \text{ on } \Gamma_0\}$ and Γ_1 satisfies a certain geometric property.

A similar result holds also in the case, where the Neumann condition on Γ_1 is replaced by the boundary condition $\partial u / \partial n + \beta u = 0$ on Γ_1 .

For the entire collection see [[Zbl 0830.00011](#)].

Reviewer: K.Pflüger (Berlin)

MSC:

[35J65](#) Nonlinear boundary value problems for linear elliptic equations

Cited in **62** Documents

Keywords:

[critical exponent](#); [existence](#)