Vancliff, M.
Quadratic algebras associated with the union of a quadric and a line in $\mathbb{P}^3$. (English)
Zbl 0837.16023
J. Algebra 165, No. 1, 63-90 (1994).

Summary: The author defines a family of graded quadratic algebras A_σ (on 4 generators) depending on a fixed nonsingular quadric Q in $\mathbb{P}^3$, a fixed line L in $\mathbb{P}^3$ and an automorphism $\sigma \in \text{Aut}(Q \cup L)$. This family contains $\mathcal{O}_q(M_2(\mathbb{C}))$, the coordinate ring of quantum 2×2 matrices. Many of the algebraic properties of A_σ are shown to be determined by the geometric properties of $\{Q \cup L, \sigma\}$. For instance, when $A_\sigma = \mathcal{O}_q(M_2(\mathbb{C}))$, then the quantum determinant is the unique (up to a scalar multiple) homogeneous element of degree 2 in $\mathcal{O}_q(M_2(\mathbb{C}))$ that vanishes on the graph in $\mathbb{P}^3 \times \mathbb{P}^3$ of $\sigma|_Q$ but not on the graph of $\sigma|_L$. Following results of *M. Artin, J. Tate*, and *M. Van den Bergh* ["The Grothendieck Festschrift", Prog. Math. 86, 33-85 (1990; Zbl 0744.14024); and Invent. Math. 106, 335- 388 (1991; Zbl 0763.14001)], we study point and line modules over the algebras A_σ, and find that their algebraic properties are consequences of the geometric data. In particular, the point modules are in one-to-one correspondence with the points of $Q \cup L$, and the line modules are in bijection with the lines in $\mathbb{P}^3$ that either lie on Q or meet L. In the case of $\mathcal{O}_q(M_2(\mathbb{C}))$, when q is not a root of unity, the quantum determinant annihilates all the line modules $M(l)$ corresponding to lines $l \subset Q$; the determinant generates the whole annihilator for such $l \subset Q$ if and only if $l \cap L = \emptyset$.

MSC:

16S35	Twisted and skew group rings, crossed products
14A22	Noncommutative algebraic geometry
17B37	Quantum groups (quantized enveloping algebras) and related deformations
14H37	Automorphisms of curves
16W50	Graded rings and modules (associative rings and algebras)
14M07	Low codimension problems in algebraic geometry

Cited in **2** Reviews
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Keywords:

graded quadratic algebras; nonsingular quadrics; coordinate rings of quantum 2×2 matrices; quantum determinants; point modules; line modules

Full Text: DOI