Stepanov, Vladimir D.
Integral operators on the cone of monotone functions. (English) Zbl 0837.26011
J. Lond. Math. Soc., II. Ser. 48, No. 3, 465-487 (1993).

Let $1 < p < \infty$, let f and v be measurable nonnegative functions on $[0, \infty)$ with v locally integrable and let

$$I(f) = \sup_{0 \le g \downarrow} \left(\int_0^\infty f g dx \right) \left(\int_0^\infty \left(\frac{1}{x} \int_0^x g dy \right)^p v(x) dx \right)^{-1/p}.$$

The main result is the sharp two-sided estimate of $I(f)$.

It allows to reduce some inequalities for non-increasing functions to modified inequalities for arbitrary measurable functions. In particular, this approach allows to find for the case in which $0 < p \le q < \infty$, $q \ge 1$, necessary and sufficient conditions on nonnegative measurable functions w and v for the following inequality

$$\left(\int_0^\infty \left(\frac{1}{x} \int_0^x g dy \right)^q w(x) dx \right)^{1/q} \le C \left(\int_0^\infty \left(\frac{1}{x} \int_0^x g dy \right)^p v(x) dx \right)^{1/p}$$

to be valid for all non-increasing nonnegative functions g with C independent of g.

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MSC:

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