Klassen, Matthew J.; **Schaefer, Edward F.**
Arithmetic and geometry of the curve $y^3+1=x^4$. (English) Zbl 0838.14018
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We consider both the arithmetic and geometry of the curve in the title. Let two points of a curve be equivalent if the image of their difference in the Jacobian of the curve has finite order. An equivalence class is called a torsion packet. The Weierstrass points form a torsion packet and they are exactly the $\mathbb{Q}(\zeta_{12})$-rational points on this curve. The latter result is obtained from the fact that the Mordell-Weil group of the Jacobian over the field $\mathbb{Q}(\zeta_{12})$ is finite. Since the Mordell-Weil group over the rationals is also finite, we can describe all solutions of the equation in fields of degree 3 or less over the rationals. In addition, we find bases for the 2- and 3-torsion of the Jacobian and describe an isogeny from the Jacobian to the product of three CM elliptic curves. The finiteness of the Mordell-Weil group was shown using a 3-descent on the Jacobian that did not make use of this isogeny.

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MSC:

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14H40 Jacobians, Prym varieties
11D25 Cubic and quartic Diophantine equations
14H55 Riemann surfaces; Weierstrass points; gap sequences

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Jacobian variety; quartic curve; Weierstrass points; Mordell-Weil group; isogeny; CM elliptic curve

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