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Frames and stable bases for shift-invariant subspaces of $L_2(\mathbb{R}^d)$. (English) Zbl 0838.42016

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Summary: Let X be a countable fundamental set in a Hilbert space H, and let T be the operator

$$T : \ell_2(X) \to H : c \mapsto \sum_{x\in X} c(x)x.$$

Whenever T is well-defined and bounded, X is said to be a Bessel sequence. If, in addition, ran T is closed, then X is a frame. Finally, a frame whose corresponding T is injective is a stable basis (also known as a Riesz basis).

This paper considers the above three properties for subspaces H of $L_2(\mathbb{R}^d)$, and for sets X of the form

$$X = \{\varphi(\cdot - \alpha) : \varphi \in \Phi,\ \alpha \in \mathbb{Z}^d\},$$

with Φ either a singleton, a finite set, or, more generally, a countable set. The analysis is performed on the Fourier domain where the two operators TT^* and T^*T are decomposed into a collection of simpler "fiber" operators. The main theme of the entire analysis is the characterization of each of the above three properties in terms of the analogous property of these simpler operators.

MSC:

42C15 General harmonic expansions, frames
47A15 Invariant subspaces of linear operators

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