Fonseca, I.; **Gangbo, W.**

Local invertibility of Sobolev functions. (English) Zbl 0839.30018

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The authors study local invertibility of mappings $v : \Omega \to \mathbb{R}^n$, where $\Omega \subset \mathbb{R}^n$, v belongs to $W^{1,n}(\Omega, \mathbb{R}^n)$, and $\det \nabla v(x) > 0$ for a.e. $x \in \Omega$. It is known (due to Reshetnyak) that such mappings are continuous in Ω and differentiable (in the classical sense, not just approximately differentiable) at almost every point of Ω. One of the main results of the paper is, roughly speaking, that if x_0 is a point of differentiability of v and $\det \nabla v(x_0) > 0$, then v is "almost invertible" in a neighborhood of x_0, in the sense that there exists an "inverse mapping" w which is defined on a suitable open neighborhood D of $v(x_0)$, belongs to $W^{1,1}(D, \mathbb{R}^n)$ and, up to sets of measure zero, satisfies $w \circ v = \mathrm{id}$ in a neighborhood of x_0 and $v \circ w = \mathrm{id}$ in D.

Moreover, the usual formulae relating the derivatives of w and v are valid. (For related results see, for example, the following papers: *J. Heinonen* and *P. Koskela*, "Sobolev mappings with integrable dilatation", Arch. Ration. Mech. Anal. 125, 81-97 (1993; Zbl 0792.30016), *J. Manfredi* and *E. Villamor*, "Mappings with integrable dilatation in higher dimensions", Bull. Am. Math. Soc. 32, No. 2, 235-239 (1995), and other papers quoted in the paper under review.)

In the second part of the paper, the authors apply the results about local invertibility to questions regarding weak lower-semicontinuity of certain non-standard variational functionals arising in models of certain materials.

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MSC:

30C65 Quasiconformal mappings in $\mathbb{R}^n$, other generalizations
49J45 Methods involving semicontinuity and convergence; relaxation

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local invertibility

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