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On a new inequality suggested by the study of certain epidemic models. (English)

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G. Gripenberg [Q. Appl. Math. 39, 317-327 (1981; Zbl 0476.92017)] studied the qualitative behavior of solutions of the equation

$$x(t) = k\left(p(t) - \int_0^t A(t-s)x(s)ds\right)\left(f(t) + \int_0^t a(t-s)x(s)ds\right). \tag{1}$$

This equation arises in the study of the spread of an infectious disease that does not induce permanent immunity. Gripenberg studied the existence of a unique bounded, continuous and nonnegative solution of (1) for $t \in \mathbb{R}_+ = [0, \infty)$ under appropriate assumptions on A and a and also obtained sufficient conditions for the convergence of the solution to a limit when $t \to \infty$. Aside from various physical meanings of the functions arising in Eq. (1), we believe that equations like (1) are of great interest in their own right and that further investigation of the qualitative behavior of their solutions even under the usual hypotheses on the functions in (1) is much more interesting.

Over the years integral inequalities have become a major tool in the analysis of various integral equations that occur in nature or are built by man. Although a great many papers have been written on various types of integral inequalities, it seems that the bounds provided by the existing results on integral inequalities do not apply directly to the study of the qualitative behavior of the solutions of Eq. (1). This amounts to finding a suitable inequality in order to achieve a diversity of desired goals. The aim of the present paper is to investigate a new integral inequality and apply it to the study of the qualitative behavior of the solutions of Eq. (1). The discrete analogue of the main result is also given.

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