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Completely monotone generalized Mittag-Leffler functions. (English) Zbl 0843.60024

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Summary: The generalized Mittag-Leffler function

$$F_{\alpha,\beta}(t)=\Gamma(\beta)\sum_{k=0}^{\infty}\frac{(-t)^k}{\Gamma(\alpha k+\beta)},\qquad t\geq 0,\quad \alpha>0,\quad \beta>0,$$

is shown to be completely monotone iff the parameters α and β satisfy $0<\alpha\leq 1$, $\beta\geq\alpha$. As $F_{\alpha,\beta}(0)=1$, the if-part is equivalent to the statement that $F_{\alpha,\beta}$ is the Laplace transform of a probability measure $\mu_{\alpha,\beta}$ supported by $\mathbb{R}_+$ (Bernstein's theorem). Apart from the trivial case $\alpha=\beta=1$ these measures are absolutely continuous with respect to the Lebesgue measure, and explicit representations of the associated densities are obtained.

MSC:

60E99 Distribution theory
33C99 Hypergeometric functions
60A99 Foundations of probability theory

Keywords:

Bernstein's theorem; Laplace transform of a probability measure