Okonek, Christian; **Teleman, Andrei**

The coupled Seiberg-Witten equations, vortices, and moduli spaces of stable pairs. (English)

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Recently, Seiberg and Witten introduced new invariants of 4-manifolds which are defined by counting solutions of certain nonlinear differential equations [*E. Witten*, Monopoles and four-manifolds, Math. Res. Lett. 1, No. 6, 769-796 (1994)]. In the present paper, the authors generalize the Seiberg-Witten equations by coupling them to connections in unitary vector bundles. More precisely, fix a Spin^c-structure on a Riemannian 4-manifold X, and denote by $\sum^{\pm}$ the associated spinor bundles. The equations, considered in the paper, are

$$\begin{cases} \mathbb{D}_{A,b}\psi = 0 \\ \Gamma(F^+_{A,b}) = (\psi\overline{\psi})_0. \end{cases}$$

This is a system of equations for a pair (A, ψ) consisting of a unitary connected in a unitary bundle E over X, and a positive spinor $\psi \in A^0(\sum^+ \otimes E)$. The symbol b denotes a connection in the determinant line bundle of the spinor bundles $\sum^{\pm}$ and $\mathbb{D}_{A,b} : A^0(\sum^+ \otimes E) \to A^0(\sum^= \otimes E)$ is the Dirac operator obtained by coupling the connection in $\sum^+$ defined by b (and by the Levi-Civita connection in the tengent bundle) with the variable connection A in E. These equations specialize to the original Seiberg-Witten equations if E is a line bundle. The main result of the paper is the following: Let (X, g) be a Kähler surface of total scalar curvature σ_g, and let $\sum$ be the canonical Spin^c-structure with associated Chern connection c. Fix a unitary vector bundle E of rank r over X, and define $\mu_g(\sum^+ \otimes E) = (\deg_g(E)/r) + \sigma_g$. Then for $\mu_g < 0$, the space of solutions of the coupled Seiberg-Witten equations is isomorphic, as a real analytic space, to the moduli space of stable pairs of TOP type E with parameter $(-1/4\pi)\sigma_g$.

Reviewer: A.Cavicchioli (Modena)

MSC:

57N13	Topology of the Euclidean 4-space, 4-manifolds (MSC2010)	Cited in **2** Reviews
57R15	Specialized structures on manifolds (spin manifolds, framed manifolds, etc.)	Cited in **10** Documents

Keywords:

Seiberg-Witten equations; 4-manifolds; Kähler surfaces; spin structures; vector bundles; moduli spaces

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