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Nonlinear oblique boundary value problems for two dimensional Hessian and curvature equations. (English) Zbl 0847.35053

Martin, Gaven (ed.) et al., Proceedings of the miniconference on analysis and applications, held at the University of Queensland, Brisbane, Australia, September 20-23, 1993. Canberra: Australian National University, Centre for Mathematics and its Applications. Proc. Cent. Math. Appl. Aust. Natl. Univ. 33, 225-237 (1994).

The first part of the paper is a survey of results concerning nonlinear oblique boundary value problems for the Hessian and the curvature equations in plane convex domains. The Hessian equation discussed is of the type $F(D^2u) = g(x,u)$. Here $F(D^2u) \equiv f(\lambda(D^2u))$, where $\lambda(D^2u)$ denotes the vector of eigenvalues of the Hessian matrix D^2u and f is a smooth real valued symmetric function in a suitable region $\Gamma \subset \mathbb{R}^2$. In the curvature equation the eigenvalues of D^2u are replaced by the principal curvatures of the graph of u. The boundary conditions imposed on $\partial\Omega$ are of the type $b(x,u,Du) = 0$. All functions f, g and b are subject to suitable restrictions. In particular, f must be positive, strictly increasing with respect to each eigenvalue λ_1 and λ_2, concave in $\Gamma+$ and vanishing on $\partial\Omega$. $g = g(x,z)$ must be increasing with respect to z. b must essentially be of the form $b = D_\gamma u + \phi(x,u,\delta u)$, where γ denotes the inner unit normal vector to $\partial\Omega$, δu denotes the tangential gradient of u relative to $\partial\Omega$, $\phi(x,z,p)$ is strictly decreasing with respect to z and negative for large z.

Under the previous hypotheses and some additional conditions, various results of existence and uniqueness are discussed for several oblique boundary value problems. The second part of the paper is devoted to a priori estimates of the second derivatives of a solution u for the Hessian and for the scalar curvature equation.

For the entire collection see [Zbl 0816.00015].

Reviewer: G.Porru (Cagliari)

MSC:

35J65 Nonlinear boundary value problems for linear elliptic equations
53C21 Methods of global Riemannian geometry, including PDE methods; curvature restrictions

Keywords:

Monge-Ampère equations; Hessian equation; existence and uniqueness; a priori estimates of the second derivatives; scalar curvature equation