

Mitidieri, Enzo

Nonexistence of positive solutions of semilinear elliptic systems in \mathbb{R}^N . (English)

Zbl 0848.35034

Differ. Integral Equ. 9, No. 3, 465-479 (1996).

The author studies nonexistence results for the elliptic system

$$-\Delta u = f(x, u, v), \quad -\Delta v = g(x, u, v) \quad \text{in } \mathbb{R}^N,$$

where $N \geq 3$ and f and g are nonnegative continuous functions. The main results are related to right-hand sides of the following type:

(i) $f(u, v) \geq a_{11}u^k + a_{12}v^p$, $g(u, v) \geq a_{21}u^q + a_{22}v^s$ for $u, v \geq 0$ with $f(0, 0) = g(0, 0) = 0$ and constants $a_{ij} > 0$ and $k, p, q, s > 1$.

(ii) $f(u, v) = u^k v^p$, $g(u, v) = u^q v^s$ with $0 \leq k, s \leq 1$ and $p, q > 1$.

(iii) $f(x, v) = a(|x|)v^p$, $g(x, u) = b(|x|)u^q$, where $p, q > 1$ and the functions $a, b \in C^1([0, \infty))$ satisfy $a(r), b(r) > 0$, $(a(r)r^\gamma)'$, $(b(r)r^\gamma)' \geq 0$ for $r > 0$, and $\lim_{r \rightarrow \infty} a(r)r^\gamma = \lim_{r \rightarrow \infty} b(r)r^\gamma = \infty$ with $\gamma = (2(p+1)(q+1) - N(pq-1))/(p+q+2)$.

Assume that the constants k, p, q, s satisfy

$$\frac{N-2}{2} \leq \max\left\{\frac{1}{k-1}, \frac{p+1}{pq-1}, \frac{1}{s-1}, \frac{q+1}{pq-1}\right\}$$

in the case (i) and

$$\frac{N}{2} < \min\left\{\frac{(p+1)(q+1) + k(q+1) - sk}{pq - (1-s)(1-k)}, \frac{(p+1)(q+1) + k(q+1) + s(p+1)}{pq + k(q+1) + s(p+1) - 1}\right\}$$

in the case (ii). Then it is proved that the elliptic system has no positive classical solutions in the first case and no positive radial solutions of class $C^2(\mathbb{R}^N)$ in the remaining cases (ii) and (iii).

In the last section, the author gives some nonexistence results for positive solutions of the above system in a smooth bounded domain with boundary data $u = v = 0$. Here it is not assumed that the system has Lagrangian structure.

Reviewer: R.Beyerstedt (Aachen)

MSC:

35J45 Systems of elliptic equations, general (MSC2000)

35J60 Nonlinear elliptic equations

Cited in **131** Documents

Keywords:

[nonexistence results](#)