**Ali, Rosihan M.**; **Singh, Vikramaditya**
**Convexity and starlikeness of functions defined by a class of integral operators.** (English)
Zbl 0851.30005
Complex Variables, Theory Appl. 26, No. 4, 299-309 (1995).

Authors abstract: For $\Lambda : [0,1] \to \mathbb{R}$ real-valued monotone decreasing function on $[0,1]$ satisfying $\Lambda(1) = 0$, $t\Lambda(t) \to 0$ as $t \to 0+$ and $t\Lambda'(t)/(1-t^2)$ increasing on $(0,1)$, we show that $M_\Lambda(f) \geq 0$ for $f$ close-to-convex where

$$M_\Lambda(f) = \inf_{|z|<1} \int_0^1 \Lambda(t) \left[ \mathrm{Re} f'(zt) - \frac{1-t}{(1+t)^3} \right] dt.$$

This is analogous to a recent result of *R. Fournier* and *St. Ruscheweyh* [Rocky Mt. J. Math. 24, No. 2, 529-538 (1994; Zbl 0818.30013)]. Analogously we obtain the least value of $\beta$ so that for $g$ analytic in $|z| < 1$, $g(0) = g'(0) - 1 = 0$, $\mathrm{Re}[e^{i\alpha}(g'(z) - \beta)] > 0$, $\beta < 1$, the functions

$$F_1(z) = z_2F_1(1, a; a+b; z) * g(z), \quad 0 < a < 1,\ b > 2$$

and

$$F_2(z) = \frac{(1-\alpha)(3-\alpha)}{2} \int_0^1 t^{-(\alpha+1)}(1-t^2)g(tz)dt, \quad 0 \leq \alpha < 1$$

are convex. Here ${}_2F_1$ is the Gaussian hypergeometric function. These results are extended to convexity and order of convexity of convex combinations of the form $\rho z + (1-\rho)F(z)$, $\rho < 1$. Corresponding starlikeness results in loc. cit. are also extended to such convex combinations.

Reviewer: K.J.Wirths (Braunschweig)

**MSC:**

30C45 Special classes of univalent and multivalent functions of one complex variable (starlike, convex, bounded rotation, etc.)



**Keywords:**

Gaussian hypergeometric function

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