Bebernes, J.; **Galaktionov, Victor A.**

On classification of blow-up patterns for a quasilinear heat equation. (English) Zbl 0851.35057

Differ. Integral Equ. 9, No. 4, 655-670 (1996).

The authors study the asymptotic blow-up behavior of nonnegative solutions to the quasilinear heat equation

$$(u)_t = (u^2)_{xx} + u^2 \quad \text{for } x \in \mathbb{R},\ t > 0, \tag{1}$$

with nonnegative, bounded, continuous initial data $u_0(x)$ in $\mathbb{R}$. Theu give a complete classification of all possible types of blow-up behavior for compactly supported initial data. The main theorems are:

Theorem 1. Let u_0 be continuous and compactly supported. Assume that u_0 intersects the level T^{-1} exactly at two points. Then there exists a constant $a_0 = a_0(u_0) \in \mathbb{R}$ such that as $t \to T^{-1}$ $(T-t)u(x,t) \to f_*(x - a_0)$ uniformly in $x \in \mathbb{R}$, where $f_*(y) = \{(4/3\cos^2(y/4)$ for $|y| < 2\pi$, 0 for $|y| \geq 2\pi\}$.

Let $I_f(t)$ be the number of intersections (in x) of the solutions $u(x,t)$ and the flat solution $u_f(t) = (T-t)^{-1}$ with the same blow-up time $T = T(u_0)$ for a fixed $t \in (0,T)$. They classify the blow-up patterns in the following theorem.

Theorem 2. Let u_0 be continuous, compactly supported and suppose

$$\lim_{t \to T^{-1}} I_f(t) = 2k$$

holds. Then there exist k numbers, $a_1 < a_2 < \cdots < a_k$, with $a_{i+1} - a_i \geq 4\pi$, such that as $t \to T$ for $i = 1, 2, \ldots, k$ $(T-t)u(x,t) \to f_*(x - a_i)$ uniformly on $|x - a_i| \leq 2\pi$ and $(T-t)u(x,t) \to 0$ uniformly on $\mathbb{R} \setminus \bigcup_{(i)} \{|x - a_i| \leq 2\pi\}$.

They also study the asymptotic blow-up behavior of nonnegative solutions of (1) when the initial data $u_0(x)$ is neither flat nor bell-shaped.

Reviewer: A.Tsutsumi (Osaka)

MSC:

35K55 Nonlinear parabolic equations
35K65 Degenerate parabolic equations
35B40 Asymptotic behavior of solutions to PDEs

Keywords:

quasilinear heat equation; blow-up behavior