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Vanishing cycles for formal schemes. II. (English) Zbl 0852.14002

Invent. Math. 125, No. 2, 367-390 (1996).

Let k be a complete discrete valuation field and k^0 its ring of integers. In part I of this work [ibid. 115, No. 3, 539-571 (1994; Zbl 0791.14008)], the author constructed and studied the vanishing cycles functor for formal schemes of locally finite type over k^0. In this part II the construction is extended to a broader class of formal schemes that includes, for example, formal completions of the above formal schemes along arbitrary subschemes of their closed fibres. The main result is a comparison theorem which states that if $\mathcal{X}$ is a scheme of finite type over a Henselian discrete valuation ring with the completion k^0 and $\mathcal{Y}$ is a subscheme of the closed fibre $\mathcal{X}_s$, then the vanishing cycles sheaves of the formal completion $\widehat{\mathcal{X}}_{/\mathcal{Y}}$ of $\mathcal{X}$ along $\mathcal{Y}$ are canonically isomorphic to the restrictions of the vanishing cycles sheaves of $\mathcal{X}$ to the subscheme $\mathcal{Y}$. In particular, the restrictions of the vanishing cycles sheaves of $\mathcal{X}$ to $\mathcal{Y}$ depend only on $\widehat{\mathcal{X}}_{/\mathcal{Y}}$, and any morphism $\varphi : \widehat{\mathcal{X}}'_{/\mathcal{Y}'} \to \widehat{\mathcal{X}}_{/\mathcal{Y}}$ induces a homomorphism from the pullback of the restrictions of the vanishing cycles sheaves of $\mathcal{X}$ to $\mathcal{Y}$ to those of $\mathcal{X}'$ to $\mathcal{Y}'$. – One also proves that, given $\widehat{\mathcal{X}}_{/\mathcal{Y}}$ and $\widehat{\mathcal{X}}'_{/\mathcal{Y}'}$, one can find an ideal of definition of $\widehat{\mathcal{X}}'_{/\mathcal{Y}'}$ such that if two morphisms $\varphi, \psi : \widehat{\mathcal{X}}'_{/\mathcal{Y}'} \to \widehat{\mathcal{X}}_{/\mathcal{Y}}$ coincides modulo this ideal, then the homomorphisms between the vanishing cycles sheaves induced by φ and ψ coincide.

These facts generalize results of part I as well as results of *G. Laumon* ["Charactéristique d'Euler-Poincaré et sommes exponentielles" (Thèse, Université de Paris-Sud, Orsay 1983)], and the author ["Vanishing cycles for non-Archimedean analytic spaces", J. Am. Math. Soc. 9, No. 4, 1187-1209 (1996)], where certain cases when $\mathcal{Y}$ is a closed point of $\mathcal{X}_s$ were considered. The main new ingredient in the proof of the comparison theorem is the recent stable reduction theorem of *A. J. de Jong* ["Smoothness, semi-stability and alterations" (preprint 1995)]. Furthermore, one proves a vanishing theorem which states that the q-dimensional étale cohomology groups of certain analytic spaces of dimension m are trivial for $q > m$. This class of analytic spaces induces, for example, the finite étale coverings $\Sigma^{d,n}$ of the Drinfeld half-plane Ω^d [*V. G. Drinfel'd*, Funct. Anal. Appl. 10, 107-115 (1976); translation from Funkts. Anal. Prilozh. 10, No. 2, 29-40 (1976; Zbl 0346.14010)].

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MSC:

14F20	Étale and other Grothendieck topologies and (co)homologies
14C25	Algebraic cycles
18F20	Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)
14G20	Local ground fields in algebraic geometry
14F99	(Co)homology theory in algebraic geometry

Keywords:

vanishing cycles functor; formal schemes; comparison theorem; vanishing theorem

Full Text: DOI EuDML